A METHOD

IN

PRIMARY ARITHMETIC.

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INTRODUCTION.

As number is the measure of quantity, number finds its clearest expression and most fitting illustration in processes of measurement. This book undertakes to set forth a method, based on measuring, through which number is taught, and a scientific and rational beginning is made in the study of arithmetic. It may be called not inappropriately the laboratory method.

Pupils are led gradually into processes of *actual measurement* and *comparison* of *actual* quantities. The importance of actual measurement and comparison of real quantities throughout is emphasized. This is not only the theory of the method, but it is the thing deemed most essential in the effort to realize the fullest and best results.

Excessive use of the measures and material need not be feared. Far less harm will result from use *beyond what is necessary* than from *too little use of these objects.* The pupil will not be hampered in his efforts to visualize or image quantity or estimate quantity relations by the presence and even excessive use of the implements of measurement. The mind will naturally free itself from these sense-objects at the proper time. The power to image will grow with the use of the measures and measuring.

The lazy teacher or the one more interested in preserving her youth and beauty, if she chance to possess these qualities, than in the welfare of the children, is in danger of making *too little* use of the illustrative material or of practically undervaluing this feature of the work.

"*Doing*" is the watchword of success, and the danger of *under* doing is much greater than the danger of *over* doing. The only thing that need be especially guarded is the movement and quantity of work, so that there will be no waste of time. A slow and unskillful teacher may cross the danger line here.

The sensuous effect supposed to result from an over-abundant use of objects and so much feared by the speculative, is nothing more than a scarecrow. There is absolutely no danger with this character of object work of such a result. A little reflection ought to satisfy anyone that this is true. It will not do to array the psychological objections to the *old kind of object work* with the *old kind of objects* against this new method.

It will be noticed that the object lessons (the developing or laboratory work) are continued throughout the course. But the objects (measures and materials) are continually varied, new work introduced from time to time, and always carefully graded to the ability and advancement of the pupils.

After thorough *presentation* and *development*, there should be whatever abstract drill is necessary for accuracy and facility. As to the degree of accuracy and facility essential in the mechanical processes, opinions differ somewhat. It is desirable that much of such work should become almost automatic during the course in primary grades; but there is no question, taking the schools of the country over, that however great the deficiency in the mere execution of processes, it is still greater from the thought or content point of view. The time allotted to the subject of arithmetic should not be given up to *figure* manipulation or *figure* juggling. This is not arithmetic, but is merely incidental to it. Real arithmetic is not addition, subtraction, multiplication or division, but it is, that certain numbers *must be* added, subtracted, multiplied or divided; it is the mental or abstract valuation of quantity—the

measuring of quantity. To perceive which of these operations should be employed and to what end, and to perceive the relations involved in measuring processes, is to grasp that *content* of arithmetic problems which alone insures the interest and pleasure that overcomes all obstacles and results in the highest degree of success. The principles of arithmetic thought are, therefore, of primary importance and should not be made subordinate to that which is in its very nature secondary and incidental. It is a startling fact that those pupils who are drilled almost exclusively upon the so-called "fundamental rules" are no stronger or better in mechanical computation in the fourth year and after than those having had the training herein indicated, and are not nearly so strong in thought power. This shows that "*figuring*" may be acquired as easily and as well without making it the conspicuous and paramount aim of all or nearly all the early lessons in arithmetic, and there is unquestionably a distinct loss in doing so.

The author is a firm believer in the analytic and inductive method of teaching arithmetic. Arithmetic *to rules* and *including* rules, but not *by rules*, is the idea. The *principles* of arithmetic are the golden nuggets, and to put pupils in possession of these is to teach them arithmetic. All arithmetic is the same and should be taught, whether mental or written, in the same general way. The only evidence that a pupil understands a problem is his explanation of it. The clearer his *insight*, the better his explanation. Let the pupil have definite and exact mathematical notions; let him express these notions *accurately* and *well*. It will be a training of inestimable value in both thought and language. It is not always a good thing to interrupt a pupil to criticise his language, but it is *essential* that he be *held to correct expression*.

Formal analysis should not be required before the second year, and then, at first, only of such questions as are clearly within the pupil's ability,—such questions as he

may answer promptly and correctly without analysis. The pupil's explanation should not be burdened with every little detail that it is possible to give. He should touch the *high points*, omitting those that may be *taken for granted* or that are *implied* in other *statements*. The thought should not be covered up with a multiplicity of words. The explanation should be brief and direct. Wordy explanations should be discouraged.

There is danger of being too formal. In general there should be as much freedom of expression as is consistent with the essential features of a good analysis. It is of little importance how the pupil *begins* the analysis or how he *ends* it, or whether he has put in the requisite number of *sinces* and *hences*, if only he has been direct, clear, concise, coherent, and grammatical. An analysis is somewhat like a rule; it is something to *work out* and to *work to*, rather than to work *by*, at least, so far as form is concerned.

It is possible for a pupil to acquire, with a little helpful drill, *forms* of *analyses* for different types of problems, and yet have very little knowledge of the mathematics of the problems. In fact, this describes a condition that is not only possible, but that is very common. Both pupil and teacher are deceived and when real and adequate tests are applied, if ever they are, there is one inevitable result, and that is *failure*; and neither can tell why. It is no wonder the examination is such a revelation. It is no wonder that teachers recoil from examinations and pupils enter them with fear and trembling.

Let pupils get the meaning of instruction. Every precaution should be taken and every proper test made *to know* that the instruction is addressed to the understanding of the pupil. Form without substance is not less fatal in education than it is in religion.

It is claimed that the character of the work here presented is such as to remove or remedy many of the evils

of primary instruction in arithmetic. Chief among its foundation principles are:—

1. The instruction is consistent with the essential idea of number; namely, that number is the measure of quantity.

2. The lessons contain arithmetic thought. They do not consist simply of hollow, mechanical, or symbol drills.

3. The object lessons, involving the use of the measures and measuring material, bring the work clearly and easily within the comprehension of the children.

NOTES TO TEACHERS.

1. The aim of this book is to present a new method of teaching primary arithmetic. The Class Lessons are intended to be, in the main, the questions, directions, and explanations of the teacher. These lessons are presented largely in question form, because this is believed to be at once the clearest, simplest, and most definite, for the character of subject-matter with which it deals. The answers present no difficulty and are therefore generally omitted, but sometimes the *form* of answer is a matter of importance. *To show the desired form*, answers have been inserted where thought necessary.

Both questions and answers often contain expressions that may appear to the teacher as odd or awkward, but the thought can hardly be expressed in phrase more apt and euphonious.

2. *Notes*, *Remarks*, *Suggestions*, *Directions*, and a few of the *parenthetical expressions* in the text are intended *for the teacher*. These explanations, if carefully observed, will enable her to advance safely and satisfactorily.

3. Before undertaking the work outlined, it is necessary that the teacher provide herself with the measures. Other material will be needed, but this can be obtained as the work advances.

For the first grade there should be as many one-foot measures and as many yard-measures as there are pupils; also a set of liquid measures, consisting of one gallon, two half-gallon, four quart, and eight pint measures, and

a set of dry measures, consisting of one bushel, two half-bushel, four peck, and eight quart measures.

For the second grade, the same material will be required as in the first grade, and in addition, a good pair of scales with a full set of weights.

The third grade will require a box of 1-inch cubes. Of these, there should be not less than one hundred.

4. The work in fractions need not be carried to the extent suggested in the course. No more should be attempted than the pupils can accomplish easily and naturally.

5. No work, whether in integers or fractions, is finished until carried through the final or abstract stage.

PART FIRST.

CHAPTER 1.

General Suggestions—Oral Work—Written Work—Signs—One, Two, and Three.

It is not expected that each of the following lessons will be completed in a single recitation, or even two or three recitations. The *time* required will depend on the conditions found to exist in each class and school.

The first lesson cannot in a very strict sense be called a number lesson. It is more nearly a reading lesson. The ability to recognize and use each of the six words which it contains, together with the ability to *write* them, must be acquired, so several lessons in preparation for this *one* will be necessary with beginners.

The same is true in a greater or less degree of each of the other lessons. Lessons *similar to those given* and as many as may be necessary should be formed by the teacher. "Make haste slowly." It pays to begin well. The object of these lessons is not so much to teach number as to get ready to teach number.

In a sense, the pupils may be said to possess all the number knowledge the first lessons contain, nevertheless they must acquire certain forms of thought and forms of expression and the meaning of certain symbols, and with these they must be made familiar, as without them progress is impossible.

Do not fall into the error that some *knowing* ones have made in assuming that children know all about *one*, and *two*, and maybe *three*, when they start to school. It does

not follow that because they know *something* about these numbers that they know all about them. If number is the measure of quantity—and I assume that it is—how can the child know *one* in any other sense than the sense of oneness when he does not know two, three, four, five, etc., and one half, one third, one fourth, one fifth, etc., and therefore cannot measure *one* by them? Not only is it not true that he does not know *one* at the beginning of his first year at school, but he does not know it at the end of the first year. It is no less erroneous to suppose that *one* can be exhausted or rather mastered fully before going to two, or two before going to three, etc.

Number involves *two* quantities, not *one* only—a *unity* to be measured and a measuring unit—so whenever the pupil has been taught a *new* number, he is able to make new comparisons, new measurements, and thus his field of number calculations is continually enlarging.

ORAL WORK.

In this preparatory work, a great number of oral drill exercises, such as follow, should be devised by the teacher. Let the work be *objective*. Such objects as pencils, books, caps, flowers, marbles, knives, etc., using no more than *three*, should be employed, and these exercises should always precede the desk work leading up to it. Let one of the pupils stand before the class with a pencil in each hand. The teacher may say, show me *one pencil*, requiring the pupil to hold up one pencil. She may then say, show me another *one pencil*, expecting the pupil to hold up the pencil in the other hand. Then she may ask: "One pencil and one pencil are how many pencils?" The pupil replies: "One pencil and one pencil are two pencils." As he does so he holds up one pencil, then the other, and places them together. His statement and illustration should be simultaneous, showing at the proper time each point in the

statement. Similarly, the idea of subtraction should be illustrated.

Let these exercises continue until all the pupils in the class can make *prompt* and *definite* answers and *correct* illustrations. Vary the objects, but always use such as can be seen and *handled* in these preliminary oral lessons. For rapid review and test work, any familiar or *well* known object may be used whether present or not.

The best *form* of statement for the beginner, in a subtraction problem, is this: "Two pencils taken from three pencils leave one pencil." Later the word, *taken*, may be dropped and it may be omitted altogether from written work. These simple explanations will serve to indicate the *reading* and interpretation of the signs of addition and subtraction which are introduced in written work from the beginning.

The signs of multiplication and division are not brought in at the very first, but the *ideas* of these two processes are. Ask such questions as "How many *ones* of pencils in two pencils?" The written form of the first of these two questions is 2 pencils ÷ 1 pencil = ? (Oral Form: In two pencils, how many ones of pencils?) This reading and form must be adhered to.

For multiplication, ask such questions as: "How many *times* one pencil in two pencils?" If there is any difficulty, lead up to it in this way: Find *one pencil* in these two pencils. Find *another* one in these two. How many times did you find one pencil in two pencils? Then two pencils are how *many times* one pencil? Written form: 1 pencil × 2 = __ pencils.. (Oral Form: Two times 1 pencil are 2 pencils.) In question form it should be read: "Two *times* 1 pencil are how many pencils?"

WRITTEN WORK.

Using marks (1 1), dots (● ●), squares (□ □ □), letters (o o) (a a), figures (2 2), etc., form lessons for desk work similar to those which follow. The characters are easily formed, requiring little practice, hence this may constitute the very first of the written exercises (desk work). This class of objects should be confined to this preparatory work, the purpose of which is to formulate and find expression for what children already know.

OUTLINE FOR FIRST YEAR.

First month, teach one, two, and three.
Second month, review and teach three and four.
Third month, review and teach halves and thirds.
Fourth month, review and teach five and fourths.
Fifth month, review and teach six and seven.
Sixth month, review and teach fifths and eight.
Seventh month, review and teach nine and eights.
Eighth month, review and teach ten.
Ninth month, review and teach ninths and tenths.

THE SIGNS.

Read:

+ and.

− taken from, beginning with the number on its right.

= are, with +.

= leaves, with −.

× times, beginning with the number on its right.

÷ how many (?) in (?) in connection with =, beginning with the number on its right. Or,

÷ (?) is how many times (?), beginning with the number on its left.

The teacher should place the following lessons and those suggested above (written work) on the blackboard and have pupils copy them:—

Lesson 1—Desk Work.

Copy: one, two, three hat, cap, cat
one, two, three hat, cap, cat
one, two, three hat, cap, cat.

Lesson 2—Desk Work.

Copy: one hat, one cap three hats
one cat, two hats three cats
two caps, two cats three caps.

Lesson 3—Desk Work.

Copy:

1, 2, 3	1, 2, 3	1, 2, 3
1 hat	1 cap	1 cat
2 hats	2 caps	2 cats
3 hats	3 caps	3 cats.

Lesson 4—Desk Work.

Copy: and, + , =, are.
1 hat and 1 hat are 2 hats.
1 hat + 1 hat = 2 hats.
1 cap and 1 cap are 2 caps.
1 cap + 1 cap = 2 caps.
1 cat and 1 cat are 2 cats.
1 cat + 1 cat = 2 cats.

Lesson 5—Desk Work.

Copy: and, +, =, are, is.
(taken) from, leaves, 0.
1 hat + 1 hat = 2 hats.
1 hat from 2 hats leaves 1 hat.
2 hats − 1 hat = 1 hat.

1 cap + 1 cap = 2 caps.
1 cap (taken) from 2 caps leaves 1 cap.
2 caps − 1 cap = 1 cap.
1 cat + 1 cat = 2 cats.
1 cat (taken) from 2 cats leaves 1 cat.
2 cats − 1 cat = 1 cat.

Lesson 6—Desk Work.

Copy:		
	one pen	one peg
	two pens	two pegs
	three pens	three pegs
	1 pen	2 pegs
	1 peg	3 pens
	2 pens	3 pegs.

Lesson 7—Desk Work.

Copy:			
	one	two	three
	1	2	3
	+	−	−
	are	from	and
	cup	top	pen
	pen	peg	peg.

Lesson 8—Desk Work.

Copy:		
	one cup	one top
	1 cup	1 top
	two cups	two tops
	2 cups	2 tops
	three cups	three tops
	3 cups	3 tops.

LESSON 9—DESK WORK.

Copy: one pen and one pen are two pens.
1 pen + 1 pen = 2 pens.
1 cup + 1 cup = 2 cups.
1 peg + 1 peg = 2 pegs.
2 pegs − 1 peg = 1 peg.
2 pens − 1 pen = 1 pen.
2 cups − 1 cup = 1 cup.
1 top + 1 top = 2 tops.
2 tops − 1 top = 1 top.

LESSON 10—DESK WORK.

Copy: 1 top + 1 top + 1 top = 3 tops.
2 tops + 1 top = 3 tops.
3 tops − 1 top = 2 tops.
3 tops − 2 tops = 1 top.
1 pen + 1 pen + 1 pen = ___ pens.
2 pens + 1 pen = ___ pens.
3 pens − 2 pens = ___ pen.
3 pens − 1 pen = ___ pens.

NOTE.—Review the foregoing lessons, leaving the answers to be filled in by the pupils as in the last four examples in lesson 10.

CHAPTER II.
Review—Three—Four.

LESSON 11—CLASS WORK.

This is a foot measure; it is used to find the length of things. Anything just as long as this measure is one foot long. Draw a line on the blackboard just the length of this measure; another; another. Count them. How many have you drawn? How long is each one? How many are one line and one line and one line?

Erase one line. How many remain?

One line taken from three lines leaves how many lines?

Erase another. How many remain? One line taken from two lines leaves how many lines? Erase another. How many remain? One line taken from one line leaves how many?

REVIEW.—How many lines were drawn on the board? (Ans. One line and one line and one line.) Tell me in another way. (Ans. There were three lines drawn on the board.) One line taken from three lines leaves how many lines? One line taken from two lines leaves how many lines? One line taken from one line leaves how many lines?

Let me see in how many ways you can group these lines (Ans. ||| $\equiv$ ∇.) How many lines in the first group? How many in the second? How many in the third? How many threes are there?

One group of lines and one group of lines and one group of lines are how many groups of lines? Erase one group. How many remain? One group taken from three groups leaves how many groups? Erase another group. One group taken from two groups leaves how many groups? Erase another. One group taken from one group leaves how many?

LESSON 12—CLASS WORK.

NOTE.—The class work for this lesson should be a review of the previous lesson together with such work as will enable the children to do the desk work at the end of the lesson. The teacher should dictate different parts of the work and have some pupil place it on the blackboard before the class.

DESK WORK.

SUGGESTIONS.—Each pupil should have a one-foot ruler. The length of the lines in this desk work may be one inch. Mark their rulers plainly at the one-inch point and show the children how to draw the lines just that length. Do not tell them how long the *board* lines are, but ask them to tell you whether they are one foot long or more or less than one foot when they recite the lesson.

Copy: 1, 2, 3, +, =.

| + | + | = | | |.

— + — + — = —+—+—.

\ + / + — = ▽.

Copy again.

LESSON 13—CLASS WORK.

NOTE.—The class work is the same as that in Lesson 12, and the lines in the desk work the same length.

DESK WORK.

Copy: 1, 2, 3, +, =.

| | | + = + ▽ = | | | = ▽.

| | + | = | | |.

△ + | | | + = = △ | | | =.

= (are), = (leaves).

Lesson 14—Class Work.

Suggestions.—On the day before this lesson is given the teacher should direct each one of the class to bring her something from home just one foot long; as, a string, a ribbon, a switch. If she prefer, she can herself provide this material The objects are: first, to impress the actual value of the foot and, second, to furnish illustrative objects.

Place together in a straight line on the table, in plain view of the class, any three articles of a kind; as, three pieces of ribbon. The objects should be changed with every question, either by the teacher or some pupil, to show the correct answers. This may seem tedious for awhile, but it pays in the end.

A ribbon one foot long and a ribbon one foot long make a ribbon how long? A ribbon one foot long and a ribbon one foot long and a ribbon one foot long make a ribbon how many feet long? A ribbon two feet long and a ribbon one foot long make a ribbon how long? Show this.

A ribbon one foot long taken from a ribbon three feet long leaves a ribbon how many feet long?

A ribbon two feet long taken from a ribbon three feet long leaves a ribbon how many feet long? A ribbon one foot long taken from a ribbon two feet long leaves a ribbon how many feet long? A ribbon two feet long and a ribbon one foot long make one how long?

Remarks.—Here the yard measure may be introduced and the class shown that just three feet of ribbon make one yard of ribbon.

So proceed with pieces of cord, sticks, etc., ending with the *use* of the yard measure.

Desk Work.

1 foot + 1 foot = ? feet.
1 foot + 1 foot + 1 foot = ? feet.
1 foot + 2 feet = ? feet.
3 feet − 1 foot = ? feet.
3 feet − 2 feet = ? feet.
2 feet − 1 foot = ? foot.
2 feet + 1 foot = ? feet.

Copy: 1, 2, 3, 4, +, −, =, ft.

LESSON 15—CLASS WORK.

NOTE.—Illustrate with objects so that the pupils may be able to write out the desk work required below. Have them read 3 — 1 = 2 as, one taken from three leaves two

DESK WORK.

1 + 1 = ?	3 − 2 = ?	3 − 3 = ?
1 + 1 + 1 = ?	2 − 1 = ?	1 − 1 = ?
1 + 2 = ?	2 − 2 = ?	2 + 1 = ?
3 − 1 = ?	1 + 2 = ?	3 ÷ 1 = ?

LESSON 16—CLASS WORK.

SUGGESTIONS.—To the following questions the answers will be only approximately correct, numerically. When the child designates an object as being a foot in length, the teacher should direct the attention of the class to the object and see that all form an estimate of its length. After this is done the teacher should apply the measure, allowing the class to see how nearly right the answer was; then, insist upon a definite and correct statement: as, the slate, or other object, is just one foot long, or, the slate is a little less than a foot long; or, it is a little more than a foot long.

See what there is in the room about a foot long.

See what you can find three feet long.

See what you can find one yard long.

See what you can find two feet long.

Straighten your arms forward and place your hands so that there will be just one foot between the tips of your fingers.

In the same way show two feet.

Straighten your arms so as to show three feet.

DESK WORK.

Copy:	1 foot = 1 ft.	1 + ? = 3
	2 feet = 2 ft.	3 − 1 = ?
	3 feet = 3 ft.	2 − 2 = ?
	3 feet = 1 yd.	2 − ? = 3
	1 + ? = 2	2 ÷ 1 = ?

LESSON 17—CLASS WORK.

Find three yard measures. Place them together in a straight line.

REMARK.—Such work should always be done before and in full view of the class.

How long is this line? (Ans. The line is three yards long.) Break the line in two places without breaking the measures. How many parts are there? How long is each part? (Ans. Each part is one yard long.) Are the parts of the same length? Are the parts of equal length? Two of these parts form a line how long?

Measure and cut just three yards of twine from this ball. Two boys may hold it up so that the class can see.

Children, how long is this string? I will cut off one yard of it; one yard taken from three yards of string leaves how many yards of string? (Ans. One yard of string taken from three yards of string leaves two yards of string.)

I will cut off another yard; one yard of string taken from two yards of string leaves how many yards of string? (Ans. One yard of string taken from two yards of string leaves one yard of string.)

Now, if I take another yard of string, how many yards will be left? (Ans. There will be no yards left.) Then, one yard of string taken from one yard of string leaves how many yards of string? (Ans. One yard of string taken from one yard of string leaves no yards of string.)

Place the yard measures side by side. Let them stand up straight so that the whole class can see them all.

I will take away one at a time; as I do so you may tell what you see. (Ans. One yard measure taken from three yard measures leaves two yard measures; one yard measure taken from two yard measures leaves one yard meas-

ure; one yard measure taken from one yard measure leaves no yard measures.)

REMARK.—In the same manner a number of pupils should be called upon to recite Use three foot measures in the same way.

NOTE.—Draw a line three yards long and show the yards with short perpendicular lines. Erase one yard at a time and require pupils to recite as above, with necessary changes in words. Always call things by their right names. Draw a line three feet long and use it in the same way.

DESK WORK.

Copy: 1, 2, 3, 4, 5, ft., yd.
1 yard + 1 yard = ? yards.
1 yard + 1 yard + 1 yard = ? yards.
2 yards + 1 yard = ? yards.
3 yards − 2 yards = ? yards.
3 yards − 1 yard = ? yards.
2 yards − 1 yard = ? yard.
1 yard + 2 yards = ? yards.

LESSON 18—CLASS WORK.

NOTE.—In the preceding lessons linear measure has been employed to teach the number *three*. In like manner *four* should be taught, the teacher constructing suitable class and desk work, modeling the lessons after those just passed over.

In the following lessons the study of *four* is continued through the *use* of liquid measure.

These are called measures. This is a gallon measure. This, a quart measure; and this, a pint measure.

What are these called?

What is this one? And this one? And this one?

If this one is filled full it will hold just one gallon. If this one is filled full it will hold just one quart. If this one is filled full it will hold just one pint.

How much will this one hold? What is the name of the measure?

How much will this one hold? What is the name of the measure?

How much will this one hold? What is the name of the measure?

Now, measure four quarts of water for me. Measure one quart. Measure three quarts. Measure two quarts.

REMARK.—A full bucket of water should be provided, and as the measures are filled the water may be poured into an empty bucket.

Place four quart measures on the table in a row. Fill them all with water. Move one quart away. One quart taken from four quarts leaves how many quarts? Move another one away. One quart taken from three quarts leaves how many quarts? Move another. One quart taken from two quarts leaves how many? Move another. One quart taken from one quart leaves how many quarts?

Put all the measures back.

REMARK.—In the same way remove more than one at a time and make problems both in addition and subtraction.

Show how many *ones* in four quarts of water. Show how many *twos* in four quarts of water. Show how many *fours* in four quarts of water.

REMARK.—The ones should be shown by placing the measures in a row, leaving a space between; the twos by grouping them by twos, etc.

DESK WORK.

1 qt. + 3 qt. = ? qt.	4 qt. − 1 qt. = ? qt.
2 qt. + 2 qt. = ? qt.	4 qt. ÷ 1 qt. = ?
2 qt. + 1 qt. = ? qt.	4 qt. ÷ 2 qt. = ?
3 qt. − 2 qt. = ? qt.	4 qt. ÷ 4 qt. = ?
4 qt. − 2 qt. = ? qt.	4 qt. − 3 qt. = ? qt.

LESSON 19—CLASS WORK.

What measure is this? (Ans. Gallon measure.)

What measure is this? (Ans. Quart measure.)

Let us find out how many quarts the gallon measure will hold. Who will try it?

NOTE.—Several pupils should be called on to make the experiment. As the measuring is done the class should count the quarts.

What measure is this? (Ans. Pint measure.)

John may show me how many pints a quart of water will make.

REMARK.—Others should be called on to do the same thing.

Mary may show how many pints will make a quart.

NOTE —Others should do the same. Review without use of objects.

How many pints will one quart of water make? Two quarts of water? How many twos of pints will two quarts of water make? How many twos of pints will three quarts of water make?

How many quarts will two pints of water make? How many quarts will two twos of pints make? How many quarts will three twos of pints make? How many quarts of water will four twos of pints make?

DESK WORK.

4 qt. = ? gal.	1 qt. ÷ 2's of pt. = ?
2 pt. = ? qt.	2 qt. ÷ 2's of pt. = ?
1 qt. = ? pt.	3 qt. ÷ 2's of pt. = ?
2 qt. = ? pt.	4 qt. ÷ 2's of pt. = ?

LESSON 20—CLASS WORK.

Place on the table in a row four pints of water. Remove one.

One pint of water taken from four pints of water leaves how many pints of water?

Remove three pints.

Remove two pints.

Remove four pints.

NOTE.—Each statement to be given, in form, as in Lesson 17.

One pint of water and one pint of water are how many quarts of water? Two pints of water and two pints of water are how many quarts of water? One quart of water and one quart of water are how many pints of water? One quart of water and two pints of water are how many pints of water? One quart of water and two pints of water are how many quarts of water?

1 gallon of water = how many quarts of water?

2 gallons of water = how many fours of quarts?

3 gallons of water = how many fours of quarts?

4 gallons of water = how many fours of quarts?

1 quart of water = how many pints?

2 quarts of water = how many pints?

NOTE —Do not neglect to illustrate.

DESK WORK.

4 pt. − 1 pt. = ? pt.	2 pt. + 2 pt. = ? qt.
4 pt. − 2 pt. = ? pt.	1 qt. + 1 qt. = ? pt.
4 pt. − 3 pt. = ? pt.	1 qt. + 2 pt. = ? pt.
1 pt. + 1 pt. = ? qt.	1 qt. + 2 pt. = ? qt.

LESSON 21—CLASS WORK.

Place on the table four quart measures of water.

REMARK —Let it be again shown how many quarts of water make a gallon.

I have one quart of water; bring me enough more to make one gallon. (Ans. One quart of water and three quarts of water are one gallon of water.)

Now, I have three quarts of water; bring me enough more to make one gallon.

NOTE.—Ask other similar questions.

Place on the table four pint measures of water.

REMARK.—Let it be again shown how many quarts of water four pints of water will make, after which have the measures filled and replaced.

I have one pint of water; bring me enough more to make two quarts of water.

I have three pints of water; bring me enough more to make two quarts of water.

I have two pints of water; bring me enough more to make two quarts.

DESK WORK.

1 qt. + ? qt. = 1 gal.	2 pt. + ? pt. = 2 qt.
3 qt. + ? qt. = 1 gal.	1 pt. + 1 pt. = ? qt.
2 qt. + ? qt. = 1 gal.	2 pt. + 1 qt. = ? pt.
1 pt. + ? pt. = 2 qt.	1 qt. + 1 qt. = ? pt.
3 pt. + ? pt. = 2 qt.	2 qt. − 1 pt. = ? pt.

CHAPTER III.

Halves—Thirds—Review.

LESSON 22—CLASS WORK.

This is what measure? (Ans. A gallon measure.)

These two are half-gallon measures; each one holds just one half gallon when full.

Let us find how many half gallons in one gallon. Frank may fill the gallon measure with water. Daisy may find how many half gallons can be made from the one gallon that Frank has given us.

REMARK —When this is done, the statement is elicited that one gallon of water makes just two half gallons of water.

Several should be called on to do the same, the teacher taking care that the class are closely observing the work.

Let us now find how many gallons of water two half gallons will make. Here are two half-gallon measures. How much will this one hold? How much will this one hold? Julia may fill both with water. How many half gallons of water are now on the table?

Henry may fill the gallon measure from the half-gallon measures. How many half gallons of water did it take, Henry, to make one gallon of water?

How many quarts will one half gallon of water make? How many quarts will the other half gallon make?

Are the two halves the same size? Is one half of anything always just as big as the other half?

How much is a half quart of water? Show me. (Ans. A half quart of water is one pint of water.)

How much is left in the measure? (Ans. One half quart.)

See whether that is a pint too. (Ans. It is just a pint.)

How many pints make a quart?
How many half quarts make a quart?
How many halves in a whole quart?
How many halves will a whole quart make?

NOTE.—Review without objects.

Show me ½ of 2 pt.
Show me ½ of 2 qt.
Show me ½ of 2 marbles.
Show me ½ of 2 gal.
Show me ½ of 4 qt.
Show me ½ of 4 pt.
Show me ½ of 4 gal.
Show me ½ of 2 ft.

DESK WORK.

½ of 2 pt. = ? pt.
½ of 1 qt. = ? pt.
½ of 4 qt. = ? pt.
½ of 1 gal. = ? qt.
½ of 4 yd. = ? yd.
½ of 1 gal. = ? pt.
½ of 4 ft. = ? ft.
½ of 2 ft. = ? ft.
½ of 2 yd. = ? yd.
½ of 4 pt. = ? qt.

LESSON 23—CLASS WORK.

In 1 gallon how many ½ gallons?
In 1 foot how many ½ feet?
In 2 gallons how many ½ gallons?
In 2 feet how many ½ feet?
In 1½ gallons how many quarts?
In 1½ gallons how many ½ gallons?
In 1½ feet how many ½ feet?
In 1½ quarts how many pints?
In 1½ quarts how many ½ quarts?

Find ½ of 3 gal. Find ½ of 3 qt. Find ½ of 3 ft.

In 2 apples how many ½ apples?
In 1 apple how many ½ apples?
In 1½ apples how many ½ apples?
In 2 of anything how many halves?
In 1 of anything how many halves?
In 1½ of anything how many halves?
2 times ½ gal. are how many gal.?
1 time ½ gal. are how many gal.?

2 times 2 gal. are how many gal.?
2 times 1 gal. are how many gal.?
2 times 1 gal. are how many $\frac{1}{2}$ gal.?
2 times $1\frac{1}{2}$ gal. are how many $\frac{1}{2}$ gal.?

DESK WORK.

$1 \div \frac{1}{2} = ?$	$\frac{1}{2}$ gal. $\times$ 2 = ? gal.
$2 \div \frac{1}{2} = ?$	$\frac{1}{2}$ gal. $\times$ 1 = ? gal.
$1\frac{1}{2} \div \frac{1}{2} = ?$	2 gal. $\times$ 2 = ? gal.
$\frac{1}{2} \div \frac{1}{2} = ?$	1 gal. $\times$ 2 = $\frac{?}{2}$ gal.
$2 = \frac{?}{2}$	$1\frac{1}{2}$ gal. $\times$ 2 = ? gal.
$1 = \frac{?}{2}$	2 gal. $\times$ 2 = $\frac{?}{2}$ gal.
$1\frac{1}{2} = \frac{?}{2}$	2 pt. $\times$ 2 = $\frac{?}{2}$ qt.

LESSON 24—PRACTICAL PROBLEMS.

1. A milkman had 1 gallon of milk and sold 1 quart; how many quarts of milk were left?

2. A little girl had a quart of milk and gave away 1 pint; how many pints were left?

3. A man had 1 gallon of cider and sold 2 quarts; how many quarts were left? How many pints?

4. A child had 2 quarts of milk. She gave 1 pint to her brother and 1 pint to her sister and then drank 1 pint; how much was left?

5. If one quart of vinegar is worth 1 nickel, how many nickels are 3 quarts worth?

6. If a pint of peanuts is worth 2 cents, what is a quart worth?

7. If a quart of nuts is worth 1 nickel, what is 1 gallon of nuts worth?

8. If 1 gallon of gasoline is worth 1 dime, how many dimes are 4 gallons worth?

9. If a pint bottle of medicine is worth $1, what is 1 quart-bottle worth? 2 quart-bottles?

DESK WORK.

1 gal. − 1 qt. = ? qt.	1 qt. − 1 pt. = ? pt.
1 gal. − 2 qt. = ? pt.	1 qt. × 4 = ? qt.
2 qt. − 3 pt. = ? pt.	1 qt. × 2 = ? qt.
1 qt. + 2 pt. = ? qt.	1 pt. × 3 = ? pt.
2 qt. + 2 pt. = ? qt.	1 qt. × 2 = ? pt.

LESSON 25.

1. A lady had a yard of ribbon. She gave a piece 1 foot long to each of her two little girls; how much was left?

2. A lady bought 4 yards of ribbon for her two little girls. She gave one of them two yards; what did the other one get?

3. A lady bought 4 yards of ribbon for her two little girls. She gave one of them 1 yard; what did the other one get?

4. A man had 4 yards of cloth to make flags. He put 1 yard in each flag; how many flags did he make?

5. A little girl had 4 hair ribbons, each 1 yard long. She lost one, then she lost another, and then she lost another; how many were left? How long was the ribbon?

6. A house has 2 rooms downstairs and 2 rooms upstairs; how many rooms in the house?

7. A hen had a nest with 4 eggs in it. A lady used 3 of them to make a cake; how many eggs were left?

8. A little girl had 1 cent. Her brother gave her another, and her sister another, and her mother gave her one; how many were given to her? How may had she then?

9. A boy had 4 marbles. He sold 2 and lost 1; how many were left? How many were sold and lost?

10. A little girl went to the store 4 times. Each time she bought a sack of candy. How many sacks did she buy?

11. A boy went to the store two times, and each time bought 2 pounds of butter; how many pounds did he get altogether?

12. A man had 4 boys. He gave each one a nickel. How many nickels did it take?

13. A man had 2 boys. He gave each one two nickels; how many nickels did it take?

DESK WORK.

1 ft. + 2 ft. = ? yd.
2 ft. + 2 ft. = ? ft.
2 ft. + 2 ft. = ? yd. ? ft.
4 ft. − 1 ft. = ? yd.

4 ft. − 3 ft. = ? ft.
4 ft. ÷ 2 ft. = ?
4 ft. ÷ 1 ft. = ?

LESSON 26—CLASS WORK.

Make a line of 1-foot measures just 1 yard long. How many measures does it take? How long is each measure?

Now break the line in two places without breaking the measures. How many parts did you make? Are the parts all the same length? How much of a yard is one part? How much of a yard is the middle part? How much of a yard is this part?

How many thirds did we find in this yard?

Show me one third of a yard. Show me another third.

How many thirds make a yard? How long is this third? This third? This third? Are all the same length?

What is one third of a yard? (Ans. One foot.)

What are two thirds of a yard?

What are three thirds of a yard? (Ans. Three feet; or, answer, one yard.)

In four feet how many thirds of a yard?

In two feet how many thirds of a yard?

In one yard how many thirds of a yard?

In four feet how many yards and feet?

DESK WORK.

1 ft. = $\frac{?}{3}$ yd. $\quad\quad$ $\frac{1}{3}$ yd. = ? ft.

2 ft. = $\frac{?}{3}$ yd. $\quad\quad$ $\frac{3}{3}$ yd. = ? ft.

3 ft. ÷ 1 ft. = ? $\quad\quad$ $\frac{2}{3}$ yd. = ? ft.

1 yd. ÷ 1 ft. = ? $\quad\quad$ 4 ft. = ? yd. ? ft.

LESSON 27—CLASS WORK.

NOTE.—Do not neglect the *use* of the measures in class work. It is only by thorough familiarity (and this comes from *persistent use*) with these units of measure that this work can be pedagogically accomplished or successfully defended.

How many feet in one yard? One foot is how much of one yard? Two feet are how much of one yard? Three feet are how much of one yard? Four feet are how many yd.? ? ft. Four feet are how much of one yard? Four feet are how many yards? Three feet are how many yards? Two feet are how many yards? One foot is how many yards?

DESK WORK.

1 yd. ÷ 1 ft. = ? $\quad\quad$ $\frac{1}{3}$ yd. × 2 = ? ft.

1 yd. ÷ $\frac{1}{3}$ yd. = ? $\quad\quad$ $\frac{2}{3}$ yd. × 2 = ? ft.

$1\frac{1}{3}$ yd. ÷ 1 ft. = ? $\quad\quad$ 1 ÷ $\frac{1}{3}$ = ?

$1\frac{1}{3}$ yd. ÷ $\frac{1}{3}$ yd. = ? $\quad\quad$ $\frac{2}{3}$ ÷ $\frac{1}{3}$ = ?

$\frac{2}{3}$ yd. ÷ 1 ft. = ? $\quad\quad$ $\frac{1}{3}$ ÷ $\frac{1}{3}$ = ?

$\frac{2}{3}$ yd. ÷ $\frac{1}{3}$ yd. = ? $\quad\quad$ $\frac{4}{3}$ ÷ $\frac{1}{3}$ = ?

$\frac{1}{3}$ yd. ÷ 1 ft. = ? $\quad\quad$ $1\frac{1}{3} = \frac{?}{3}$

$\frac{1}{3}$ yd. ÷ $\frac{1}{3}$ yd. = ? $\quad\quad$ $\frac{1}{3} - \frac{1}{3}$ = ?

1 yd. + 1 ft. = ? ft. $\quad\quad$ 1 − $\frac{1}{3}$ = ?

$\frac{2}{3}$ yd. − 1 ft. = ? ft. $\quad\quad$ 1 − $\frac{2}{3}$ = ?

$1\frac{1}{3}$ yd. − 2 ft. = ? ft. $\quad\quad$ $\frac{2}{3} - \frac{1}{3}$ = ?

$1\frac{1}{3}$ yd. − 2 ft. = ? yd. $\quad\quad$ $1\frac{1}{3} - \frac{2}{3}$ = ?

$\frac{2}{3}$ yd. × 2 = ? yd. $\quad\quad$ $\frac{2}{3} + \frac{2}{3}$ = ?

$\frac{1}{3}$ yd. × 2 = ? yd. $\quad\quad$ $\frac{1}{3} + \frac{2}{3}$ = ?

Reviews—Fourths.

Lesson 28—Class Work.

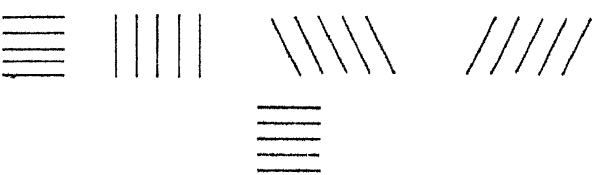
Draw a group of four lines on the board, each a foot long. I will add one line; how many can tell the number of lines there are now? How long is each line? If I should make them all into *one* line, how long would that line be?

Note.—This should be done, cutting the line into *foot lengths* by *short* perpendicular lines.

2 lines and 2 lines and 1 line are how many lines? 3 lines and 2 lines are how many lines? 1 line and 4 lines are how many lines? 3 lines and 2 lines are how many lines? How many *ones* of lines are there in five lines? (Ans. In 5 lines there are five 1's of lines.) How many 2's of lines in 5 lines? How many 3's of lines in 5 lines?

Erase one line, Mary. One line taken from 5 lines leaves how many lines? Erase another, Davie. One line taken from 4 lines leaves how many lines? How many have been erased? How many were there at first? Then 2 lines taken from 5 lines leave how many lines? Etc.

Draw four groups of 5 lines each on the board. I will



add 1 group. How many groups now? How many lines in this group? In this one? In this one? In this one?

4 groups of lines and 1 group of lines are how many groups of lines?

REMARK.—With some convenient object cover up that *part* of the number of groups you do not wish the children to see, leaving exposed to view the part you are talking about and wish them to observe.

2 groups of lines and 2 groups of lines and 1 group of lines are how many groups of lines? 2 groups of lines and 3 groups of lines are how many groups of lines? Erase one group. How many groups remain? One group taken from 5 groups leaves how many groups? Erase another. One group taken from 4 groups leaves how many?

DESK WORK.

1 ft. + 1 ft. + 1 ft. = yd.
1 ft. = $\frac{1}{3}$ yd.
2 ft. = $\frac{2}{3}$ yd.
$\frac{1}{3}$ yd. = ft.
$\frac{2}{3}$ yd. = ft.
4 ft. = yd. ft.
$\frac{4}{3}$ yd. = ft.

4 ft. ÷ $\frac{1}{3}$ yd. =
3 ft. ÷ 1 ft. =
1 yd. ÷ 1 ft. =
1 yd. ÷ $\frac{1}{3}$ ft. =
$\frac{2}{3}$ yd. ÷ $\frac{1}{3}$ yd. =
1 ft. ÷ $\frac{1}{3}$ yd. =
$\frac{1}{3}$ yd. + $\frac{1}{3}$ = yd.

LESSON 29—CLASS WORK.

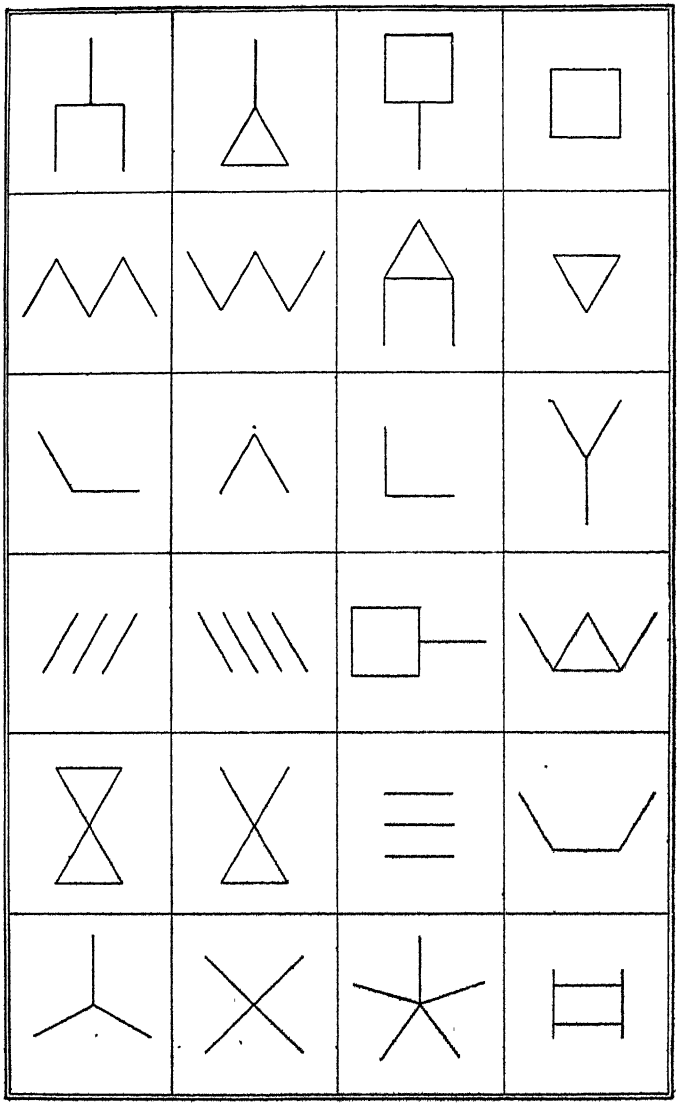
NOTE 1.—Here the term, inch, is introduced for the first time. As the numbers grow larger, there is need for the smaller units of measure, so the inch may be used to great advantage from this time on. Six inches show six units (ones) in a short and convenient space, but six yards (written work) in one length cannot be so easily used for illustrative purposes. This principle applies as well in the use of other measures.

NOTE 2.—The teachers should mark the children's rulers at the 1-inch point with a heavy ink line; and show them how to use their rulers and to draw a one-inch line.

NOTE 3.—Now substitute this new line for the one-foot line and follow the plan of the preceding lesson.

NOTE 4.—An interesting and valuable exercise may be had by placing the figures in the desk lesson following on the blackboard;

DESK WORK.



and then, pointing out rapidly, but irregularly, one figure at a time, requiring the children to give the number of lines it contains. This should be a short exercise and continue from day to day until the pupils can answer without hesitation. At first apply "*Remark*" in preceding lesson.

NOTE 5.—A most excellent drill for quick recognition of groups is to have cards with colored circles on them symmetrically arranged into groups. Show the card but a moment to the pupils and have them answer instantly how many it contains.

NOTE 6.—Make the lines in the desk work shown on the preceding page just one inch long.

LESSON 30—CLASS WORK.

NOTE.—The material suggested in Lesson 14, Chapter II. for teaching *three* and *four* should be used here in teaching five; and the work done in the same general manner.

DESK WORK.

5 ft. − 1 ft. = ft. 5 ft. − 3 ft. = ft.

5 ft. − 4 ft. = ft. 5 ft. − 2 ft. = ft.

2 ft. + ft. = 5 ft. 1 ft. + ft. = 5 ft.

3 ft. + ft. = 5 ft.

1 ft. + 1 ft. + 2 ft. = ft. 2 ft. + 2 ft. + 1 ft. = ft.

1 ft. + 1 ft. + 3 ft. = ft. 2 ft × 2 + 1 ft. = ft.

inch = in.

LESSON 31—CLASS WORK.

NOTE.—The plan of this lesson is given in Lesson 16, Chapter II. Let the first point be to review the children in finding *objects* and *distances* illustrating four-foot lengths, then let them *show* what the length of each would be if *one foot were added*, i. e., to 4 foot lengths.

This is an important lesson, and the utmost pains should be taken to follow out the suggestions.

DESK WORK.

2 ft. + 2 ft. = ft. 1 ft. + 2 ft. = ft.

4 ft. − 1 ft. = ft. 5 ft. − 2 ft. = ft.

5 ft. = yd. ft. 4 ft. + 1 ft. = yd.

1 yd. + 1 ft. = ft. 1 yd. + 2 ft. = ft.

LESSON 32—CLASS WORK.

NOTE.—The plan of this lesson is the same as that of Lesson 17, Chapter II. It should, of course, be adapted to the number in hand—five.

DESK WORK.

3 yd. + 2 yd. = yd.	2 yd. + 2 yd. = yd.
1 yd. + 4 yd. = yd.	5 yd. − 2 yd. = yd.
4 yd. + 1 yd. = yd.	5 yd. − 3 yd. = yd.
5 yd. ÷ 1 yd. =	4 yd. ÷ 2 yd. =
5 yd. ÷ 5 yd. =	3 yd. ÷ 1 yd. =
2 yd. × 2 = yd.	$1\frac{1}{3}$ yd. + $\frac{1}{3}$ yd. = yd.

LESSON 33—CLASS WORK.

NOTE.—Model this after Lesson 18, Chapter II.

NOTE.—Review the following:

Show $\frac{1}{2}$ qt. How many $\frac{1}{2}$ qt. in 1 qt.? Show $2\frac{1}{2}$ qt. Show $1\frac{1}{2}$ qt.

Measure 5 pints, using pint measures; and place them on the table.

Separate to show:—

3 pints and 2 pints.

2 pints and 2 pints and 1 pint.

3 pints and 1 pint and 1 pint.

Measure 5 pints, using quarts and pints. How many quarts and pints in 5 pints? (Ans. There are 2 quarts and 1 pint in 5 pints.) How many quarts in 5 pints? (Ans. There are $2\frac{1}{2}$ quarts in 5 pints.) Measure 5 quarts and place them on the table in a row.

NOTE.—*Remove* different numbers of quarts, calling on different pupils to *recite*.

Refill and place again to show:—

How many 1's of quarts in 5 pints?

How many 2's of quarts in 5 pints?

How many 2's of quarts in 2 quarts?

How many 2's of quarts in 1 quart?
How many 2's of quarts in 4 quarts?
How many 2's of quarts in 5 quarts?

NOTE.—The pupil should make a definite statement in answer to each question, after illustrating.

DESK WORK.

$\frac{1}{2}$ qt. = pt.	$2\frac{1}{2}$ qt. = pt.	5 pt. = qt.
4 qt. + 1 qt. = qt.	5 qt. − 3 qt. = qt.	
$1\frac{1}{2}$ qt. = pt.	5 qt. − 4 qt. = pt.	
5 qt. ÷ 1 qt. =	2 qt. ÷ 2 qt. =	
4 qt. ÷ 2 qt. =	5 qt. ÷ 2 qt. =	

LESSON 34—CLASS WORK.

SUGGESTION.—Review, by asking names of measures to be used. Model the lesson after Lesson 19, Chapter II.

Measure 5 quarts using only quart measures.
Measure 5 quarts using gallon and quart measures.
Measure 5 quarts using half-gallon and quart measures.

NOTE.—Let several pupils do this.

Show how many pints a quart of water will make.
Show how many pints 2 quarts of water will make.
Show how many pints $\frac{1}{2}$ quart of water will make.
Show how many pints $1\frac{1}{2}$ quarts of water will make.
How many 2's of pints will 1 quart of water make?
How many 2's of pints will 2 quarts of water make?
How many 2's of pints will 3 quarts of water make?
How many 2's of pints will 4 quarts of water make?
How many 2's of pints will 5 quarts of water make?
(Ans. 1 quart of water will make *one* 2 of pints.)
(Ans. 2 quarts of water will make *two* 2's of pints.)

NOTE.—Each answer should be *illustrated.*

How many quarts will *one* 2 of pints make?
How many quarts will *two* 2's of pints make.
How many quarts will *three* 2's of pints make?

How many quarts will *four* 2's of pints make?

How many quarts will *five* 2's of pints make?

(Ans. One 2 of pints will make 1 quart.)

(Ans. Two 2's of pints will make 2 quarts.)

Show how many gallons and quarts 5 quarts of water will make.

Show how many ½ gallons and quarts 5 quarts of water will make.

DESK WORK.

4 qt. = gal.	2½ qt. = pt.
5 qt. = gal.	1 qt. ÷ 2's of pt. =
2 qt. = pt.	2 qt. ÷ 2's of pt. =
1 qt. = pt.	3 qt. ÷ 2's of pt. =
½ qt. = pt.	4 qt. ÷ 2's of pt. =
1½ qt. = pt.	5 qt. ÷ 2's of pt. =

LESSON 35—CLASS WORK.

NOTE.—Model after Lesson 20, Chapter II.

Place on the table in a row 5 pints of water. Remove different numbers of pints at different times, calling on various pupils to recite.

1 pint of water and 1 pint of water are how many quarts of water?

3 pints of water and 2 pints of water are how many quarts?

2 pints of water and 2 pints of water are how many quarts?

½ quart of water is how many pints?

1 quart of water and ½ quart of water are how many pints?

1 quart of water and 1½ quarts of water are how many pints?

1 quart of water and 3 pints of water are how many quarts?

1 quart of water and 3 pints of water are how many pints?

1 gallon is how many quarts?
1 gallon is how many $\frac{1}{2}$ gallons?
$\frac{1}{2}$ gallon is how many quarts?
$\frac{1}{2}$ gallon is how many pints?
2 gallons are how many 4's of quarts?
3 gallons are how many 4's of quarts?
4 gallons are how many 4's of quarts?
5 gallons are how many 4's of quarts?
1 gallon is how many 4's of pints?
2 gallons are how many 4's of pints?

DESK WORK.

5 pt. − 1 pt. = pt.	5 pt. − 2 pt. = pt.
5 pt. − 4 pt. = pt.	5 pt. − 5 pt. = pt.
$\frac{1}{2}$ qt. = pt.	1 qt. + 1 qt. = pt.
1 qt. + $\frac{1}{2}$ qt. = pt.	1 qt. + $1\frac{1}{2}$ qt. = pt.
1 qt. + 3 pt. = pt.	3 pt. + 1 qt. = qt.
$\frac{1}{3}$ of 3 pt. = pt.	$\frac{1}{3}$ of $1\frac{1}{2}$ qt. = qt.
$\frac{1}{2}$ of 4 qt. = pt.	$\frac{1}{2}$ of 3 qt. = pt.
$\frac{1}{2}$ of 3 qt. = qt.	$\frac{1}{2}$ of 5 qt. = qt.

LESSON 36—CLASS WORK.

NOTE.—Model after Lesson 18, Chapter II.

Fill with water and place on the table the following measures: 1 gallon, 2 half gallons, 4 quarts, and 4 pints.

I have in my hand 1 quart of water, bring me enough half gallons to make 5 quarts.

Now I have $\frac{1}{2}$ gallon of water, bring me enough quarts to make 5 quarts.

I have $\frac{1}{2}$ gallon and 1 quart, bring me enough quarts of water to make 5 quarts of water.

I have 2 half gallons, bring me enough pints to make 5 quarts of water.

I have $1\frac{1}{2}$ quarts of water, bring me enough quarts of water to make 5 quarts.

I have 4 pints of water, bring me enough quarts to make 5 quarts of water.

I have 1 quart of water, bring me enough gallons to make 5 quarts of water.

Fill five pint measures of water and place them on the table.

I have 1 pint of water, bring me enough pints to make 5 pints; enough quarts.

I have 2 pints of water, bring me enough quarts to make 5 pints; enough pints.

I have 1 quart of water, bring me enough quarts to make 5 pints; enough pints.

I have 4 pints of water, bring me enough quarts to make 5 pints; enough pints.

I have 3 pints of water, bring me enough pints to make 5 pints; enough quarts.

DESK WORK.

$\frac{1}{2}$ gal. + qt. = 5 qt.	$1\frac{1}{2}$ qt. + qt. = 5 qt.
4 pt. + qt. = 5 qt.	1 qt. + gal. = 5 qt.
1 pt. + qt. = 5 pt.	3 pt. + qt. = 5 pt.
1 qt. + pt. = 5 pt.	2 qt. + pt. = 5 pt.
$1\frac{1}{2}$ qt. + pt. = 5 pt.	$1\frac{1}{2}$ qt. + qt. = 5 pt.

LESSON 37—CLASS WORK.

NOTE.—Model this lesson after Lesson 22, Chapter III, using the gallon as a unit and the quart as a fractional part. It is best to have *one gallon* measure and *four quart* measures.

NOTE.—Review the names of measures to be used.

Show first by actual and exact measuring that *four* quarts make a gallon; and then reverse and show that one gallon makes *four* quarts.

Now taking up *one* quart ask the question: How much of a gallon is *one* quart? Two quarts? Three quarts? Four quarts?

Don't fail to cover all the points in Lesson 22.

DESK WORK.

1 qt. = $\frac{1}{4}$ gal.	2 qt. = gal.	3 qt. = $\frac{3}{4}$ gal.
1 gal. = qt.	1 gal. = $\frac{1}{4}$ gal.	5 qt. = gal. qt.
5 qt. = gal.	$\frac{1}{4}$ gal. = qt.	$\frac{3}{4}$ gal. = qt.
	$1\frac{1}{4}$ gal. = qt.	

Lesson 38—Class Work.

Place on the table two half gallon measures filled with water and two empty quart measures.

How much of a gallon of water is one quart of water? (Ans. One quart of water is one fourth of a gallon of water.)

Show how many one fourths of a gallon of water one half gallon of water will make.

How many one fourths of a gallon in one half gallon? How many one fourths of a gallon in the other half gallon?

How many one fourths of a gallon in 2 half gallons? In a whole gallon?

How many halves in a dollar?

How many fourths of a dollar in a half dollar? How many fourths in a half of anything? One half is how many fourths? One half is how many times one fourth?

One half of one half of a gallon of water is how many quarts of water? Then, how many *fourths* of a gallon of water is it?

What is a half of a half of anything?

Desk Work.

1 gal. = $\frac{}{4}$ gal.		1 gal. = $\frac{}{2}$ gal.
$\frac{1}{2}$ gal. = $\frac{}{4}$ gal.		$\frac{2}{2}$ gal. = $\frac{}{4}$ gal.
$1 \div \frac{1}{2}$ =	$1 \div \frac{1}{4}$ =	$\frac{1}{2}$ of $\frac{1}{2}$ =
$\frac{1}{2}$ gal. $\times$ 2 = qt.		$\frac{1}{2}$ gal. $\times$ 2 = $\frac{}{4}$ gal.
$\frac{1}{4}$ gal. $\times$ 2 = $\frac{}{2}$ gal.		$\frac{1}{2}$ gal. $\times$ 2 = gal.

CHAPTER

Six—Seven—Rev.

LESSON 39—CLASS WORK.

Draw on the board a line one inch long. Under that, draw another the same length.

Class, how many lines are now on the board?

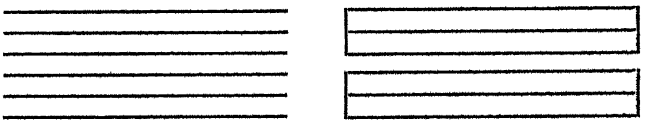
Draw three more.

How many now, class?

I will add *one* more. Now, how many, class?

Let us see how many *threes* of inch lines in six one-inch lines.

NOTE.—The teacher can easily show the groups of three by joining the ends as shown in the following:



How many *threes* of boys are six boys?

NOTE.—Illustrate.

How many *threes* of girls are six girls?

How many *threes* of children are six children?

How many *threes* of measures will six foot-measures make?

NOTE.—In the same manner show the *twos* of lines in six inch-lines, etc.

DESK WORK.

5 in. + 1 in. = in.	6 in. ÷ 2 in. =
6 in. − 1 in. = in.	4 in. ÷ 2 in. =
3 in. + 2 in. = in.	5 in. ÷ 2 in. =
3 in. + 3 in. = in.	3 in. ÷ 2 in. =
2 in. + 2 in. + 2 in. = in.	6 in. − 3 in. = in.
4 in. + 2 in. = in.	6 in. − 4 in. = in.
6 in. ÷ 3 in. =	3 in. × 2 = in.

LESSON 40—CLASS WORK.

Draw a line on the board as nearly six-inches long as you can make it. Measure and correct it.

Now, separate it in the middle. How much of the whole line is each part?

NOTE.—Better, how much of the whole line is this part? How much of the whole line is this part?

How long is this part?

How long is this part?

How many three-inch lines in a six-inch line?

What is one half of six inches?

What are two halves of six inches?

Again: Draw on the board a line six inches long. Measure it. Correct it. Now separate it two inches from one end. Separate it two inches from the other end. How many parts are there? How long is this part? This part? This part? How much of the whole line is this part? This part? This part? How many two-inch lines in a six-inch line? What is one third of six inches? Show me one third of the line. Cover up one third of the line. How many thirds can be seen? How long are two thirds of the line?

What are two thirds of six inches?

What are three thirds of six inches?

DESK WORK.

$\frac{1}{2}$ of 6 in. = in.	6 in. × $\frac{1}{3}$ = in.
$\frac{1}{3}$ of 6 in. = in.	4 in. × 1 = in.
$\frac{2}{3}$ of 6 in. = in.	4 in. × $\frac{1}{2}$ = in.
$\frac{3}{3}$ of 6 in. = in.	3 in. × 1 = in.
2 in. × 2 = in.	3 in. × $\frac{1}{3}$ = in.
3 in. × 2 = in.	3 in. × $\frac{2}{3}$ = in.
6 in. × 1 = in.	6 in. × 1 = in.
6 in. × $\frac{1}{2}$ = in.	6 in. × $\frac{2}{3}$ = in.

LESSON 41—CLASS WORK.

Draw on the board a line one foot long. Measure and correct it. Draw under this, the same length, 4 more lines. How many are now on the board? How long is each? I will add another one-foot line. How many are there now? How many *ones* of lines are there? How many *twos* of lines are there? How many *threes* of lines? How many *fours* of lines?

NOTE.—Illustrate as in last lesson.

DESK WORK.

6 ft. ÷ 1 ft. =	3 ft. + 3 ft. = ft.
6 ft. − 1 ft. = ft.	3 ft. × 2 = ft.
6 ft. − 2 ft. = ft.	2 ft. + 2 ft. + 2 ft. = ft.
6 ft. ÷ 3 ft. =	2 ft. × 3 = ft.
6 ft. ÷ 2 ft. =	6 ft. ÷ 4 ft.

LESSON 42—CLASS WORK.

Form a line, using 6 foot measures.

NOTE.—Let a number of pupils stand in front of the class to hold the measures up in a line end to end.

Break the line in the middle. How many parts are there? How long is this part? How long is this part? How many threes of feet in six feet? Three feet are how much of six feet? What is one half of six feet? What are two halves of six feet?

Place the parts together again?

Break the line two feet from this end; now, two feet from this end.

How many parts now, class? How long is this part? This part? This part? How many *twos* of feet in six feet? How many parts are there? What is one third of six feet? Two thirds of six feet?

A LESSON PLAN.

NOTE.—Let the teacher stand before the class with several measures (yard measures and foot measures) at hand, and so change and manipulate them as to *illustrate* the answer to each of the following questions. To begin, she may hold in vertical position a yard measure in each hand and ask:

What measures are these? How long is each one? How long a line of measures will they make (placing them end to end horizontally)? How many *feet* long? In six feet how many yards? In three feet (removing one measure) how many yards?

In four feet (substituting a foot measure for one of the yard measures) how many yards?

In one foot (using only a foot measure) how many yards?

In two feet (using two foot measures) how many yards?

In five feet (using a yard measure and two foot measures) how many yards?

NOTE.—The exercise just described is one to be used in the development of *any* number from this on.

Schemes to be placed on the board to remain for drill:—

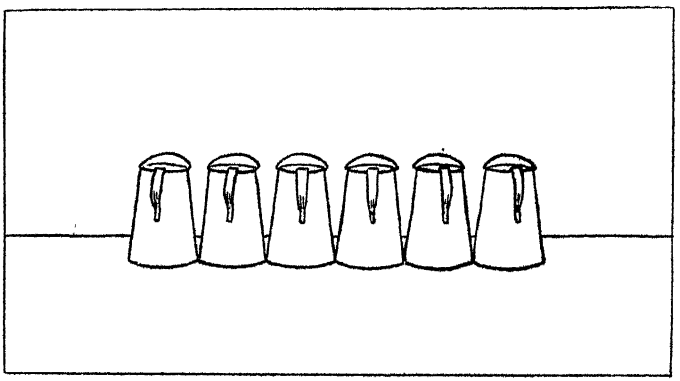
1. 6 feet 4 feet 2 feet 1 foot 5 feet 3 feet $4\frac{1}{2}$ feet $1\frac{1}{2}$ feet	=	yard	2. $1\frac{1}{3}$ yards 1 yard 2 yards $\frac{1}{3}$ yard $\frac{2}{3}$ yard $1\frac{2}{3}$ yards $1\frac{1}{2}$ yards $\frac{1}{2}$ yard	=	feet

DESK WORK.

$\frac{1}{2}$ of 6 ft. = ft.	6 ft. × $\frac{1}{3}$ = ft.
$\frac{1}{3}$ of 6 ft. = ft.	4 ft. × 1 = ft.
$\frac{1}{4}$ of 6 ft. = ft.	4 ft. × $\frac{1}{2}$ = ft.
$\frac{3}{4}$ of 6 ft. = ft.	3 ft. × 1 = ft.
2 ft. × 2 = ft.	3 ft. × $\frac{1}{3}$ = ft.
3 ft. × 2 = ft.	3 ft. × $\frac{2}{3}$ = ft.
6 ft. × 1 = ft.	6 ft. × $\frac{2}{3}$ = ft.
6 ft. × $\frac{1}{2}$ = ft.	2 ft. × $\frac{1}{2}$ = ft.

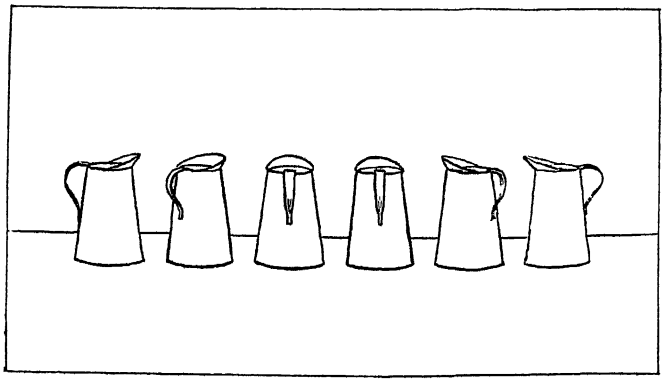
LESSON 43—CLASS WORK.

Fill six quart measures with water and place them in a row on the table touching each other.



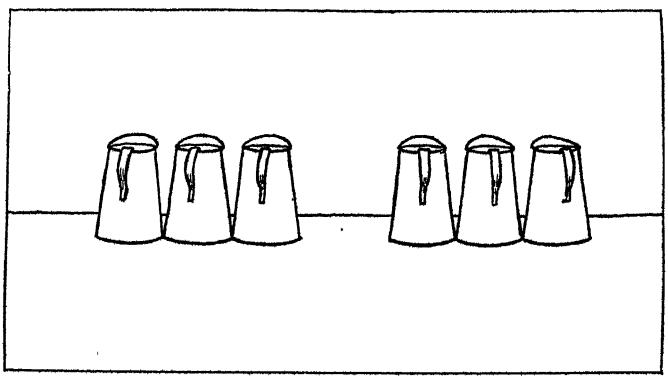
Tell me now what you see on the table. (Statement: I see a six of quarts of water on the table.)

In six quarts of water, how many *ones* of quarts?



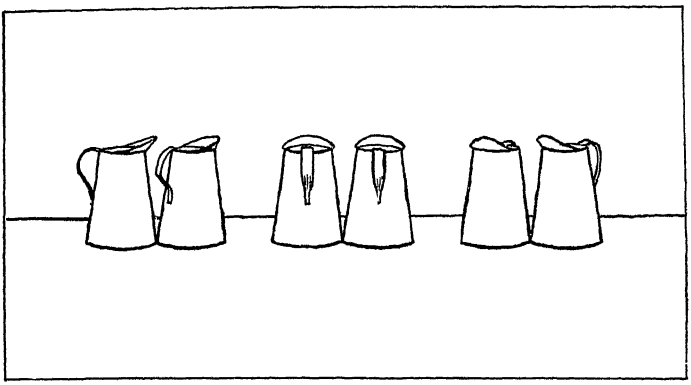
(Statement: In six quarts of water there are six ones of quarts.)

In six quarts of water, how many threes of quarts?



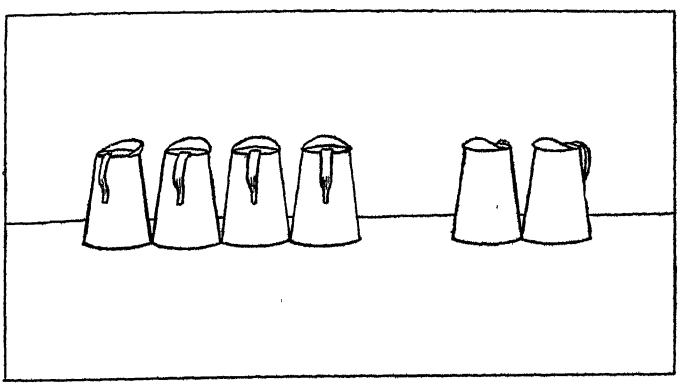
(Statement: In six quarts of water, there are *two* threes of quarts.)

In six quarts of water, how many *twos* of quarts of water?



(Statement: In six quarts of water, there are three *twos* of quarts.)

In six quarts of water, how many half-gallons of water? Then, how many gallons of water?



(Statement: In six quarts of water there are one and one half gallons of water.)

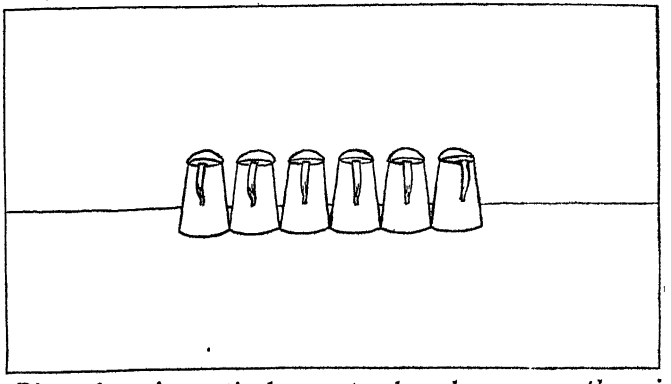
How many quarts make a gallon? How many fours of quarts in six quarts of water?

DESK WORK

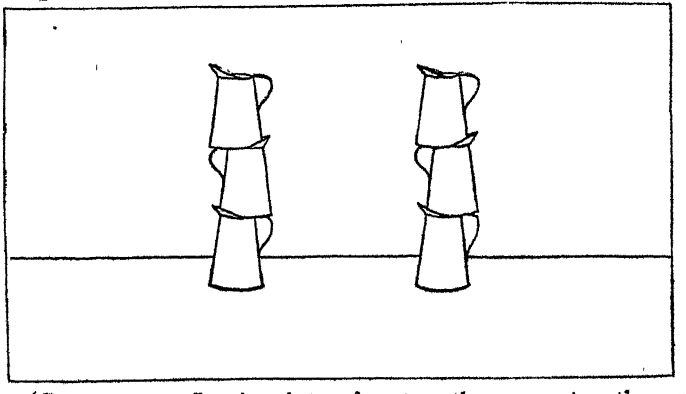
6 qt. = gal. qt.	6 qt. − 2 qt. = gal.
6 qt. = gal.	6 qt. − 4 qt. = gal.
6 qt. ÷ 2 qt. =	2 qt. × 3 = qt.
6 qt. ÷ 3 qt. =	6 qt. ÷ 4 qt. =
4 qt. + 2 qt. = qt.	6 qt. ÷ $\frac{1}{2}$ gal. =
3 qt. × 2 = qt.	$\frac{1}{2}$ of 6 qt. = qt.

LESSON 44—CLASS WORK.

Fill six pint measures and place them in a row on the table, touching each other.

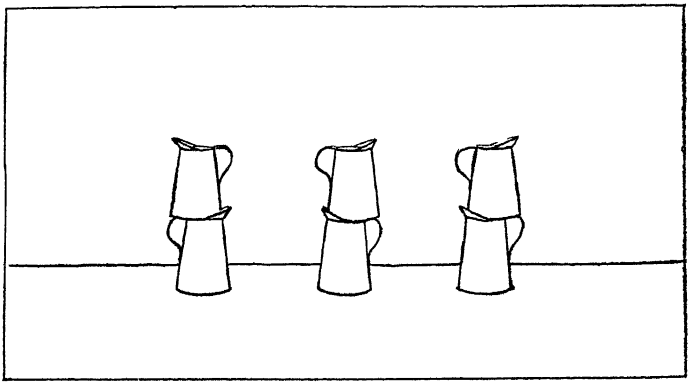


Place them in vertical rows to show how many *threes* in six pints of water.



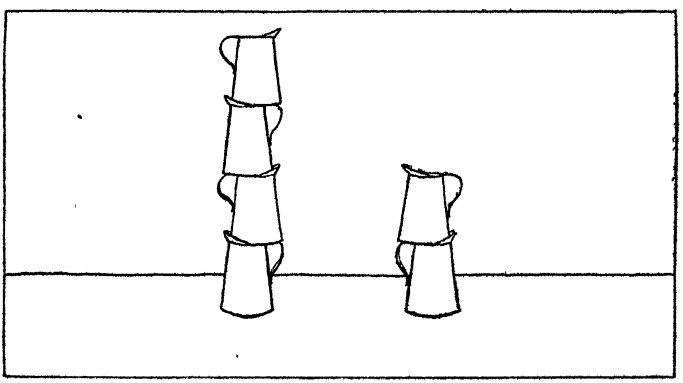
(Statement: In six pints of water, there are *two* threes of pints of water.)

Place them in vertical rows to show how many *twos* of pints of water in six pints of water.



(Statement: In six pints of water there are *three* twos of pints.) Then, how many quarts in six pints?

Place them vertically to show how many *fours* of pints of water in six pints of water.



(Statement: In six pints of water there are one and one half fours of pints of water.)

Schemes to be placed on the board *to remain* for drill:

1. 6 pints, 4 pints, 2 pints, 1 pint, 3 pints, 5 pints } = quart

3. 1 quart, 2 quarts, 3 quarts, ½ quart, 2½ quarts, 1½ quarts } = pint

2. 1 gallon, ½ gallon, ¾ gallon, ¼ gallon, 1½ gallons, 1¼ gallons } = quart

4. 4 quarts, 6 quarts, 2 quarts, 1 quart, 3 quarts, 5 quarts } = gallon

DESK WORK.

6 pt. = qt
6 pt. = ½ gal.
6 pt. = gal.
6 pt. ÷ 2 pt. =
6 pt. ÷ 3 qt. =
4 pt. + 2 pt. = qt.

5 pt. − 3 pt. = qt.
6 pt. − 4 pt. = qt.
6 pt. − 3 pt. = qt.
3 pt. × 2 = qt.
2 pt. × 3 = qt.
3 pt. + 3 pt. = pt.

LESSON 45—CLASS WORK.

Fill six pint measures with water and place them in a row on the table close together. (See Lesson 44.)

Now separate them in the middle. How many pints of water in this part? In this part? Is this part just as large as the other part? How much of the whole six pints is this part? How much of the whole is this part? What is ½ of six pints of water? What are ⅔ of six pints of water? What is ½ of six quarts of water? What is ½ of six gallons of milk? ½ of six dollars? ⅔ of six quarts of water? ⅔ of six gallons of milk? ⅔ of six dollars?

Now make *three* groups of the same size if you can. Are the groups of the same size? How do you know? How many groups are there? How much of the whole six is this one? This one? This one?

What is one third of six pints of water?
What is one third of six quarts of water?
What are two thirds of six quarts of water?
What are three thirds of six quarts of water?
What is one third of six gallons of milk?
What are two thirds of six gallons of milk?
What are three thirds of six gallons of milk?
What is one third of six dollars?
What are two thirds of six dollars?
What are three thirds of six dollars?

DESK WORK.

$\frac{1}{2}$ of 6 pt. =	pt.	$\frac{3}{3}$ of 6 pt. =	qt.
$\frac{2}{3}$ of 6 pt. =	pt.	3 pt. × 2 =	pt.
$\frac{1}{3}$ of 6 pt. =	pt.	3 pt. × 2 =	qt.
$\frac{1}{3}$ of 6 pt. =	qt.	2 pt. × 2 =	qt.
$\frac{2}{3}$ of 6 pt. =	qt.	2 pt. × 3 =	pt.
$\frac{2}{3}$ of 6 qt. =	qt.	2 pt. × 3 =	qt.
$\frac{3}{3}$ of 6 pt. =	pt.	$\frac{2}{3}$ of 6 in. =	in.

LESSON 46—CLASS WORK.

Each of you take your foot measure. Now, hold up the measure (vertically) to show six inches. How long is the part of the measure above your fingers?

Now, hold it straight in front of you (horizontally) and tell me how many six-inches in a foot.

If the measure were sawed in two at the middle, how long would each part be? Show six inches again just as you did at first. Now move your fingers down just one inch. How many inches are you showing now? If the measure were cut off there, would the parts be the same length? How long would the long part be? How long would the short part be? One inch taken from six inches leaves how many inches? Six inches and one inch are how many inches?

NOTE.—Let the teacher now furnish each child with seven inch-sticks.

I have put some little sticks on your desks; I wonder whether you can tell me how long they are. I wonder whether you can tell me how *many* there are. Count them. Put them all in a line; how long is the line? Put them in two lines; make the lines as nearly equal as you can. How many in the longer line? In the shorter? How long is the longer? How long is the shorter? Four inches and three inches make how many inches?

Now put the sticks in three lines, making them as nearly equal as you can. How many inch sticks in the first line? How many in the middle line? How many in the last line? How long is the first line? How long is the middle line? How long is the last line? Now, tell me what you have found out. (Ans. In 7 inches there are 2 inches and 2 inches and 3 inches.)

DESK WORK.

6 in. + 1 in. = in.	5 in. + 2 in. = in.
1 ft. ÷ 6 in. =	7 in. − 5 in. = in.
4 in. + 3 in. = in.	7 in. ÷ 2 in. =
6 in. − 1 in. = in.	7 in. ÷ 3 in. =
2 in. + 2 in. = in.	7 in. − 3 in. = in.
2 in. + 2 in. + 3 in. = in.	7 in. − 2 in. = in.

LESSON 47—CLASS WORK.

NOTE.—Compare Lessons 42 and 46 and form both class and desk lessons, using linear measure for objective work, to teach *seven*.

LESSON 48—CLASS WORK.

NOTE.—Compare Lessons 44 and 46 and, using liquid measure, form lessons for both class and desk work on the number *seven*.

LESSON 49—CLASS WORK.

NOTE.—Compare Lessons 43 and 46 and use liquid measure after the plan outlined in Lesson 46 for still other lessons on *seven*.

LESSON 50—CLASS WORK.

Who can name the days of the week? What is the first day of the week? What is the last day of the week? Which is the day of rest? What days do you go to school? On what days is there no school? Which are the work days?

How many days make a week? Name them in regular order.

DESK WORK

Copy twice:

Sunday, Tuesday, Thursday, Saturday.
Monday, Wednesday, Friday,

Put the school days in a group to themselves.

Put the work days in a group.

Put those that are not school days in a group.

LESSON 51—CLASS WORK.

Name the days of the week. How many days in a week? How many school days in a week? How many days are not school days? How many work days in a week? A week is equal to what two numbers of days? A week is equal to what other two numbers of days? To what other two numbers of days? To what three numbers of days? To what other three numbers of days?

John came to school three days last week; how many days was he absent?

Mary was absent one day; how many days was she at school?

NOTE.—Make other examples.

DESK WORK.

7 da. ÷ 1 da. =	6 da. ÷ 2 da. =
3 da. × 2 = da.	6 da. ÷ 3 da. =
4 da. + 3 da. = da.	½ of 6 da. = da.
7 da. − 5 da. = da.	½ of 7 da. = da.
2 da. + 2 da. + 3 = da.	⅓ of 6 da. = da.
5 da. + 2 da. = da.	⅓ of 7 da. = da.
7 da. − 4 da. = da.	2 da. × 3 + 1 da. = da.

CHAPTER VI.

Fifths—Sixths—Eight—Sevenths.

NOTES.

1. The class work is intended to prepare the pupils for the desk work.

2. It is not intended that the questions shall be "put" in the exact form in which they are given, nor always in the same order

3. Counting by single units should usually be avoided, and pupils should be led to see groups and make additions by groups.

4. The expression, *6 of feet, 4 of pints*, etc., appears awkward at first, but it in no way hampers the work and is easily dropped when there is no longer necessity for its use, so both this and its equivalent, *6-feet*, in "How many 6-feet in 12 feet?" are freely used and recommended. The reason is they express an idea of unity, of oneness, essential to the number process—the measuring process.

Lesson 52—Class Work.

Here are some pieces of money on the table. Who can point out a nickel? Who can find a cent? Who can show how many cents make a nickel?

Which would you rather have a nickel or five one-cent pieces?

How much of a nickel is a one-cent piece i. e., of the value of a nickel?

Two one-cent pieces?

Three one-cent pieces?

Four one-cent pieces?

A nickel has how many fifths in it?

A nickel has how many cents in it?
Which is worth more, a cent or one fifth of a
What is one fifth of a nickel?
What are two fifths of a nickel?
What are three fifths of a nickel?
What are four fifths of a nickel?
What are five fifths of a nickel?

DESK WORK.

5 ct. = nickel.	$\frac{1}{5}$ nickel =
1 ct. = nickel.	$\frac{4}{5}$ nickel =
3 ct. = nickel.	$\frac{3}{5}$ nickel = ct.
4 ct. = nickel.	$\frac{2}{5}$ nickel = ct.
2 ct. = nickel.	$\frac{5}{5}$ nickel = ct.
$\frac{1}{5}$ nickel − 1 ct. = ct.	1 nickel ÷ 2 ct. =

LESSON 53—CLASS WORK.

NOTE.—For this lesson and subsequent ones involving the *fathom*, a sounding line—small rope—should be provided. It should be about twenty feet long and knotted at six, twelve, and eighteen feet from one end to show one, two, and three fathoms. A good way to show the length of the *fathom* and enable the children to hold the fact clearly in mind is to have them measure off foot-lengths and wrap and tie a colored string around the line to separate the six foot-lengths of the first fathom. The *use* of the fathom should also be carefully taught.

How many would like to learn a new unit to-day? I want to talk to you about the fathom. How many ever heard of a fathom? Who can spell fathom? Who will write it on the board? Who can tell me how many feet make a fathom? What is the fathom used for? Why is it necessary to know the depth of the water? How many have ever seen a man measuring the depth of the water from the deck of a steamboat? Did he have a pole or a line? What caused the line to go to the bottom? How

many know what a sand-bar is? How many ever saw a boat aground? Is there more danger where the water is deep or where it is shallow? If the water is one *fathom* deep, how many feet deep is it? If it is a half a fathom deep, how many feet deep is it? If it is a third of a fathom deep, how many feet deep is it? If it is four feet deep, what part of a fathom deep is it? How many feet make a fathom? Suppose water is just one foot deep, what part of a fathom deep is it? If it is two feet deep? If it is three feet deep? If it is four feet deep? If it is five feet deep? If it is six feet deep? How many sixths of a fathom in a whole fathom?

One third of a fathom is how many feet? Two thirds of a fathom are how many feet? One third of a fathom is how many sixths of a fathom? Two thirds of a fathom are how many sixths of a fathom? One sixth of a fathom and one sixth of a fathom are what part of a fathom? One half of one third of a fathom is what part of a fathom? One sixth of a fathom taken from one third of a fathom leaves what part of a fathom? One third of a fathom and one sixth of a fathom are what part of a fathom? How many one sixths of a fathom in one third of a fathom?

DESK WORK.

1 f. ÷ 1 ft. =	$1\frac{2}{3}$ yd. = ft. = f.
1 f. ÷ $\frac{1}{6}$ f. =	$2\frac{1}{3}$ yd. = ft. = f.
$1\frac{1}{6}$ f. ÷ 1 ft. =	$1\frac{1}{3}$ yd. = ft. = f.
$1\frac{2}{6}$ f. ÷ 1 ft. =	1 ft. = yd. = f.
$1\frac{2}{6}$ f. ÷ 2 ft. =	2 ft. = yd. = f.
$1\frac{2}{6}$ f. ÷ 3 ft. =	5 ft. = yd. = f.
8 ft. = f. ft.	3 ft. = yd. = f.
7 ft. = f. ft.	6 ft. = yd. = f.
7 ft. = f.	4 ft. = yd. = f.
8 ft. = f.	8 ft. = yd. = f.
2 yd. + 2 ft. = f.	7 ft. = yd. = f.

LESSON 54—CLASS WORK.

SIXTHS COMPARED WITH THIRDS AND HALVES.

NOTE —Provide foot measures and half-foot measures.

Place *upon* this yard measure just enough foot measures to make one yard. How many does it take? One foot is how much of a yard? Two feet? Three feet?

Now place *upon* the yard measure just enough $\frac{1}{2}$-foot measures to make a yard. How many are required? Who can tell me how much of a yard $\frac{1}{2}$-foot is? Two $\frac{1}{2}$-feet? Three $\frac{1}{2}$-feet? Four $\frac{1}{2}$-feet? Five $\frac{1}{2}$-feet? Six $\frac{1}{2}$-feet? How many sixths in a yard? How many thirds in a yard?

Now place *upon* this foot measure just enough $\frac{1}{2}$-foot measures to make a foot. How many sixths of a yard does it take? Show me a sixth of a yard. Show me a third of a yard. How many sixths of a yard in a third of a yard?

Find a $\frac{1}{2}$-yard measure. Place on the $\frac{1}{2}$-yard measure just enough $\frac{1}{6}$-yard measures to make $\frac{1}{2}$ yard. How many does it take? How many $\frac{1}{6}$ of a yard in $\frac{1}{3}$ of a yard?

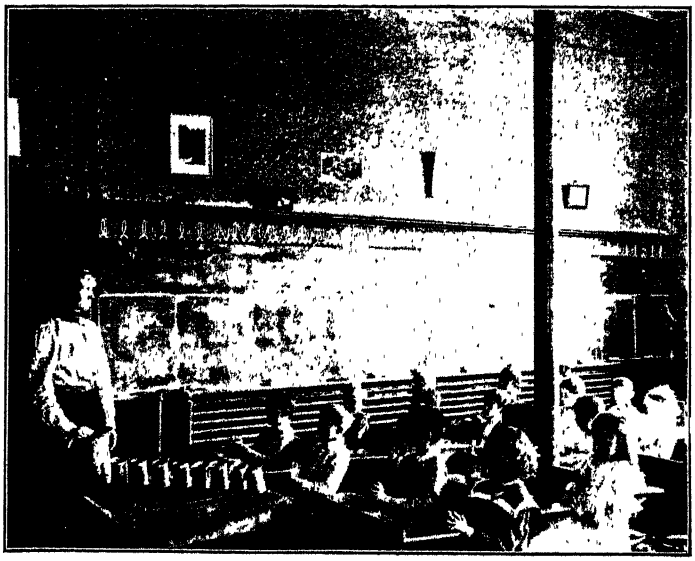
DESK WORK.

1 yd. ÷ $\frac{1}{3}$ yd. =	$\frac{1}{3}$ yd. × 2 = yd. = ft.
1 yd. ÷ $\frac{1}{2}$ yd. =	$\frac{1}{3}$ yd. × 3 = ft. = yd.
1 yd. ÷ $\frac{1}{6}$ yd. =	$\frac{2}{3}$ yd. × 3 = ft. = yd.
$\frac{1}{2}$ yd. ÷ $\frac{1}{6}$ yd. =	$\frac{1}{2}$ of 6 ft. = ft. = yd.
$\frac{1}{3}$ yd. ÷ $\frac{1}{6}$ yd. =	$\frac{1}{3}$ of 6 ft. = ft. = yd.
$\frac{2}{3}$ yd. ÷ $\frac{1}{6}$ yd. =	$\frac{1}{6}$ of 6 ft. = ft. = yd.
$\frac{1}{3}$ yd. × 2 = ft.	$\frac{2}{3}$ of 6 ft. = ft. = yd.

Schemes for blackboard:—

1. 5 cents, 1 cent, 4 cents, 3 cents, 2 cents, 6 cents, 7 cents = nickel

2. 1 nickel = $\frac{1}{5}$ nickel, $\frac{2}{5}$ nickel, $\frac{3}{5}$ nickel, $\frac{4}{5}$ nickel, $\frac{5}{5}$ nickel, $\frac{6}{5}$ nickel, $\frac{7}{5}$ nickel = cents



DIRECTION.—Using long measure and liquid measure as shown in the lesson on *six* (Chapter V.), teach the number *eight*.

LESSON 55—CLASS WORK.

Now, children, we have used two kinds of measures since we began to study numbers. One we call long measure and the other liquid measure. Who can tell me what measures belong to long measure? What is long measure used for? (Ans. To measure *length*.) Name some things the foot is used to find the length of. Name some things the yard is used to find the length of. Name some things the inch is used to find the length of. Would we find length if we measured to find how wide the blackboard is? Would we find length if we measured to find how high a boy is?

NOTE.—The term *unit* may be introduced and taught and the children led to speak of the foot-unit, the gallon-unit, etc.

A liquid is a substance that will flow. Will water flow? Do you know anything else that will flow? What do you think liquid measure is used for, then? What are the units of liquid measure? Name something the gallon is used to measure. Name something the quart is used to measure. Name something the pint is used to measure.

I wish to show you to-day some new measures—some we have not used before. Here they are: This is called a bushel measure; this a half-bushel measure; this a peck measure, and this a quart measure. Do you think any of these are made to hold water? Could we measure liquids with them? We call these dry measures. Name things the bushel is used to measure. Name things the peck is used to measure. Name things the quart is used to measure. Name things the half bushel is used to measure.

Fill a peck measure with corn. Now fill as many quart measures from the peck as you can. How many quarts of corn does the peck make? Group the quarts of corn to show how many make a peck. Group them to show how many in a half peck. How many in all? How many in this half? How many in this half? How many fours of quarts in a peck of corn? What is $\frac{1}{2}$ of a peck of corn in quarts? Group them to show how many twos of quarts in a peck of corn.

(Statement: In a peck of corn there are four twos of quarts of corn.) What is $\frac{1}{4}$ of a peck of corn in quarts? What are $\frac{2}{4}$ of a peck of corn in quarts? What are $\frac{3}{4}$ of a peck of corn in quarts? What are $\frac{4}{4}$ of a peck of corn in quarts?

Now, group the eight quarts of corn to show a peck of corn in two unequal parts. How many quarts in this part? (Ans. There are three quarts of corn in this part.) How many in this part? Then, 3 quarts of corn and 5 quarts of corn are how many quarts of corn? (Ans. 8 quarts of corn.) Give another name for it. (Ans. A peck of corn.) Make another *two* unequal groups. How many in this

group? How many in this group? 6 quarts of corn and 2 quarts of corn are how many quarts of corn? Make another *two* groups. How many in this? In this? How many in both? How much in both?

DESK WORK.

4 qt. + 4 qt. = qt.	5 qt. + 3 qt. = pk.
4 qt. × 2 = qt.	1 pk. − 4 qt. = qt.
8 qt. − 2 qt. = qt.	1 pk. − 4 qt. = pk.

2 qt. + 2 qt. + 2 qt. + 2 qt. = qt.

2 qt. × 4 = qt.	$\frac{1}{4}$ of 8 qt. = qt.
8 qt. ÷ 4 qt. =	$\frac{1}{2}$ of 8 qt. = qt.
8 qt. ÷ 2 qt. =	$\frac{1}{2}$ of 8 qt. = pk.
5 qt. + 3 qt. = qt.	$\frac{3}{4}$ of 8 qt. = qt.

LESSON 56—CLASS WORK.

NOTES.

1. Let it be shown by actual measurement how many pecks make a bushel.

2 A common tin or wooden scoop will be found very convenient in measuring.

3. Children are very slow at first in doing this kind of work, but they should be constantly directed and hurried by the teacher.

4. It is often better for the teacher to handle the measures to save time and secure definiteness and directness of work.

5. The teacher has, no doubt, before this discovered the necessity of a plain table for the measures and measuring.

6. Let the two half-bushel measures be filled with oats and one placed upon the other, so the children may get a good idea of the value of a bushel. It is well to use a bushel basket also.

7. Now, let the experiment be made to determine how many peck measures can be filled from this bushel.

How many pecks of oats do you find a bushel of oats will make?

Let us put the four pecks back into the bushel measure to see that we made no mistake. Now, how many pecks will it take to fill the half-bushel measure? To fill the other half-bushel measure? To make the whole bushel?

DESK WORK.

1 bu. ÷ ½ bu. =	1 bu. − 2 pk. = bu.
½ bu. ÷ 1 pk. =	1 bu. − 1 pk. = pk.
⅔ bu. ÷ 1 pk. =	2 pk. × 2 = bu.
2 pk. + 2 pk. = pk.	½ of 1 bu. = pk.
2 pk. + 2 pk. = bu.	½ of 4 pk. = bu.
½ bu. + 1 pk. = pk.	½ of 8 pk. = bu.

LESSON 57—CLASS WORK.

Using quart measures, measure a peck of corn. How many 8's of quarts in a peck? How many 1's of quarts in a peck? This peck measure will make how many 8's of quarts? This one? How many 8's of quarts in two pecks of corn? In three pecks of corn? In four pecks of corn? In a bushel of corn how many 8's of quarts? A peck is how many times a quart? A bushel is how many times a peck?

Using the peck measures, measure a bushel of corn. How many pecks in a bushel? One peck is how much of a bushel? Three pecks are what part of a bushel? Two pecks are what part of a bushel? Four pecks are what part of a bushel?

One half bushel of corn will make how many pecks of corn? One peck is how much of a bushel? This holds a half bushel and this a fourth of a bushel; how many fourths of a bushel of corn in a half bushel of corn?

DESK WORK.

1 pk. ÷ 1 qt. =	1 bu. ÷ 2's of pk. =
1 pk. ÷ 8's of qt. =	½ bu. ÷ 1 pk. =
2 pk. ÷ 8's of qt. =	½ bu. ÷ ¼ bu. =
3 pk. ÷ 8's of qt. =	1 bu. ÷ ½ bu. =
4 pk. ÷ 8's of qt. =	¼ of 1 bu. = pk.
1 bu. ÷ 8's of qt. =	½ of 1 bu. = pk.

2 bu. ÷ 8's of qt. = $\frac{3}{4}$ of 1 bu. = pk.
1 bu. ÷ 1's of pk. = $\frac{4}{4}$ of 1 bu. = pk.
1 bu. ÷ 4's of qt. = 3 bu. ÷ 8's of qt. =

LESSON 58—CLASS WORK.

Fill the 8 quart measures with wheat and place them in a row on the table touching. What measure will just hold these eight quarts? How many half pecks in a peck? Group them to show the quarts in a half peck. How many quarts in a half peck? How many *fours* of quarts in eight quarts? How many fourths of a peck in a peck? Group them to show the quarts in one fourth of a peck. How many quarts in one fourth of a peck? How many *twos* of quarts in a peck? What is one half of eight quarts? What is one fourth of eight quarts?

Place them in vertical rows to show how many threes of quarts in eight quarts. Place them in vertical rows to show how many fives. To show how many sixes.

DESK WORK.

1 pk. = qt.
4 qt. + 4 qt. = pk.
6 qt. + qt. 1 pk.
5 qt. + qt. 1 pk.
1 qt. × 8 = qt.
1 qt. × 8 = pk.
2 qt. × 4 = qt.

2 qt. × 4 = pk.
8 qt. ÷ 3 qt. =
8 qt. ÷ 4 qt. =
8 qt. ÷ 2 qt. =
8 qt. ÷ 5 qt. =
8 qt. ÷ 6 qt. =
8 qt. ÷ 7 qt. =

LESSON 59—CLASS WORK.

NOTE.—Review the lessons on the week. Using the week as the unit and the day as a fractional part, teach sevenths.

How many days in a week? Then, one day is what part of a week? If one day is one seventh of a week, how much are two days? Three days? Seven days? Eight days?

Are eight days more or less than a week? Eight days are how many weeks and days? Are how many weeks? Four days are how much of a week? Six days? Five days?

DESK WORK.

7 da. + 1 da. = da.
7 da. − 2 da. = da.
4 da. + 2 da. = da.
8 da. − 2 da. = da.
$\frac{1}{2}$ of 8 da. = da.
$\frac{1}{4}$ of 8 da. = da.
$\frac{3}{4}$ of 8 da. = da.
1 wk. = da.
1 da. = wk.

3 da. = wk.
2 da. = wk.
5 da. = wk.
4 da. = wk.
7 da. = wk.
8 da. = wk. da.
8 da. = wk.
6 da. = wk.
$\frac{1}{2}$ of a wk. = da.

CHAPTER VII.

Nine—Eighths—Comparison of Halves, Fourths, and Eighths—Practical Problems.

NOTE.—In the study of *nine* use first linear measure, then liquid measure, then dry measure. Refer to lessons on the development of *six*, *seven*, and *eight*. Arrange appropriate class work and desk work.

FOR THE BLACKBOARD.

Column addition.

5	2	4	1	5	2
3	3	2	3	1	2
1	2	2	3	2	2
0	2	1	2	1	2
3	2	1	3	3	4
2	0	6	3	3	4
0	4	2	2	3	1
4	3	0	1	0	0

Combinations for practice.

2	4	5	3	7	4
3	2	2	4	2	5
6	1	6	1	7	1
2	3	1	5	1	4
2	5	8	6	3	4
2	3	1	3	3	4

NOTE.—All work placed on the board for drill should be changed often and new work added as the class progresses.

LESSON 60—CLASS WORK.

Fill the gallon measure with water. With this gallon of water fill as many pint measures as possible and place them in a row on the table. How many pints of water does the gallon make? Place them close together; how much water in the eight pints of water? If I move away one pint of water how much of the whole gallon do I take away? How many pints make a gallon? One pint is what

part of a gallon? Two pints? Three pints? Four pints? Seven pints? Five pints? Six pints? Eight pints?

NOTE.—Let the answers to the questions above be in eighths only.

Place them together again. One eighth of a gallon of water taken from (removing one pint of water) eight eighths of a gallon of water leaves how many eighths of a gallon?

Two eighths of a gallon of water taken from (removing two pints) eight eighths of a gallon of water leave how many eighths of a gallon of water?

Three eighths of a gallon of water taken from (removing three pints) eight eighths of a gallon of water leave how much?

Four eighths of a gallon of water taken from (removing four pints) eight eighths leave how many eighths of a gallon of water?

Five eighths? Six eighths? Seven eighths? Eight eighths?

NOTE.—Make problems in addition also, carrying out fully the object feature in the class work.

DESK WORK.

$\frac{8}{8}$ gal. − $\frac{1}{8}$ gal. = pt.	$\frac{2}{8}$ gal. + $\frac{3}{8}$ gal. = pt.
$\frac{8}{8}$ gal. − $\frac{3}{8}$ gal. = gal.	$\frac{7}{8}$ gal. − $\frac{3}{8}$ gal. = gal
$\frac{7}{8}$ gal. − $\frac{4}{8}$ gal. = gal.	$\frac{7}{8}$ gal. − $\frac{4}{8}$ gal. = pt.
$\frac{3}{8}$ gal. + $\frac{5}{8}$ gal. = gal.	$\frac{4}{8}$ gal. + $\frac{4}{8}$ gal. = gal.
1 gal. − $\frac{5}{8}$ gal. = pt.	$\frac{2}{8}$ gal. + 6 pt. = pt.
2 qt. − $\frac{4}{8}$ gal. = qt.	5 pt. + $\frac{2}{8}$ gal. = gal.

LESSON 61—CLASS WORK.

Place on the table just $\frac{1}{4}$ of a gallon of water in one measure. Place on the table just $\frac{2}{8}$ of a gallon of water in two measures. Is the $\frac{1}{4}$ gallon more than the $\frac{2}{8}$ gallon? Is it less than the $\frac{2}{8}$ gallon? Are they just the same? How do you know they are the same?

Place on the table, using the right measure, just $\frac{1}{2}$-gallon of water. See how many *fourths* of a gallon you can make of it. How many $\frac{1}{4}$'s of a gallon of water in a $\frac{1}{2}$-gallon of water?

See how many $\frac{1}{8}$'s of a gallon of water the $\frac{1}{2}$-gallon will make. How many? Two $\frac{1}{2}$-gallons of water will make how many $\frac{1}{8}$'s of a gallon?

A gallon of water is how many $\frac{1}{8}$'s of a gallon of water? A $\frac{1}{4}$-gallon of water is how many $\frac{1}{8}$'s of a gallon of water? A $\frac{1}{2}$-gallon of water is how many $\frac{1}{8}$'s of a gallon? $\frac{1}{2}$-gallon of water and $\frac{1}{4}$-gallon are how much water? $\frac{1}{4}$-gallon of water and $\frac{1}{8}$-gallon of water are how much water? $\frac{1}{2}$-gallon of water and $\frac{1}{8}$-gallon of water are how much? $\frac{3}{4}$-gallon of water and $\frac{1}{8}$-gallon of water are how much?

DESK WORK.

$\frac{1}{2}$ gal. + $\frac{1}{4}$ gal. = gal.	$\frac{1}{2}$ gal. + $\frac{1}{8}$ gal. = gal.
$1\frac{1}{2}$ gal. + $\frac{1}{2}$ gal. = gal.	$\frac{3}{4}$ gal. + $\frac{1}{8}$ gal. = gal.
$\frac{3}{4}$ gal. + $\frac{1}{2}$ gal. = gal.	$\frac{5}{8}$ gal. + $\frac{1}{8}$ gal. = gal.
$\frac{1}{4}$ gal. + $\frac{1}{4}$ gal. = gal.	$\frac{3}{8}$ gal. + $\frac{1}{8}$ gal. = gal.
$\frac{1}{4}$ gal. + $\frac{1}{8}$ gal. = gal.	$\frac{3}{8}$ gal. + $\frac{1}{4}$ gal. = gal.

LESSON 62—CLASS WORK.

Using the proper measure, measure $\frac{1}{8}$ of a gallon of water. Using the proper measure, measure $\frac{1}{4}$ of a gallon of water. Using the proper measure, measure $\frac{1}{2}$-gallon of water. Using the pint measure, measure $\frac{2}{8}$-gallon.

Using the quart measure, measure $\frac{2}{8}$-gallon.

Using the quart measure, measure $\frac{1}{4}$-gallon.

Using the pint measure, measure $\frac{4}{8}$-gallon.

Using the quart measure, measure $\frac{4}{8}$-gallon.

Using the half-gallon measure, measure $\frac{4}{8}$-gallon.

What other name for $\frac{4}{8}$ of a gallon? Using the quart measure, measure $\frac{6}{8}$ of a gallon. What other name for $\frac{6}{8}$ of a gallon of water?

DESK WORK.

$\frac{5}{8}$ gal. $-$ $\frac{1}{8}$ gal. $=$ gal.	$\frac{3}{4}$ gal. $-$ $\frac{5}{8}$ gal. $=$ gal.
$\frac{7}{8}$ gal. $-$ $\frac{1}{8}$ gal. $=$ gal.	$\frac{1}{2}$ gal. $-$ $\frac{3}{8}$ gal. $=$ gal.
$\frac{3}{8}$ gal. $-$ $\frac{1}{8}$ gal. $=$ gal.	$\frac{3}{8}$ gal. $-$ $\frac{1}{4}$ gal. $=$ gal.
$\frac{1}{2}$ gal. $-$ $\frac{1}{8}$ gal. $=$ gal.	$\frac{7}{8}$ gal. $-$ $\frac{3}{8}$ gal. $=$ gal.
$\frac{1}{4}$ gal. $-$ $\frac{1}{8}$ gal. $=$ gal.	$\frac{5}{8}$ gal. $-$ $\frac{1}{4}$ gal. $=$ gal.
$\frac{3}{4}$ gal. $-$ $\frac{3}{8}$ gal. $=$ gal.	$\frac{7}{8}$ gal. $-$ $\frac{1}{4}$ gal. $=$ gal.

Which is more, $\frac{1}{2}$-gallon of water or $\frac{4}{8}$ of a gallon?

Which is more, $\frac{1}{2}$-gallon of water or $\frac{2}{4}$ of a gallon?

Which is more, $\frac{2}{4}$-gallon of water or $\frac{4}{8}$ of a gallon.

LESSON 63—CLASS WORK.

Place on the table $\frac{1}{2}$-gallon of water. Separate this quantity of water into two equal parts. What measures should you use? How much of the half-gallon of water is this part? This part? How much of the whole gallon is this part? This part? What is one half of one half of a gallon of water? What is one half of two fourths of a gallon of water? Place on the table one quart of water. What part of a gallon is one quart? Let us find a half of this one fourth of a gallon. Fill as many pint measures

as you can with this fourth of a gallon. How many does it fill? Show me one half of the $\frac{1}{4}$-gallon. Show me the other half of the $\frac{1}{4}$-gallon of water. How much of a whole gallon is this half? This half? Then, what is one half of $\frac{1}{4}$ of a gallon of water?

NOTE.—Teach in the same general way the half of $\frac{4}{8}$, $\frac{3}{8}$, and $\frac{2}{8}$.

Show two times $\frac{1}{8}$ of a gallon of water.
Show (by grouping) two times $\frac{2}{8}$ of a gallon of water.
Show (by grouping) two times $\frac{3}{8}$ of a gallon of water.
Show (by grouping) two times $\frac{4}{8}$ of a gallon of water.
Show (by grouping) two times $\frac{1}{4}$ of a gallon of water.
Show (by grouping) two times $\frac{2}{4}$ of a gallon of water.

NOTE.—Review without objects.

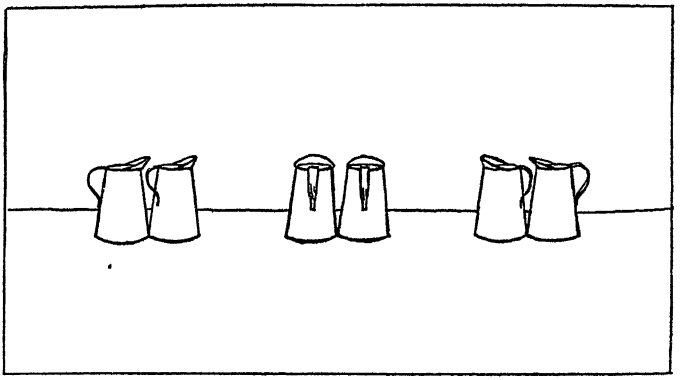
DESK WORK.

$\frac{1}{2}$ of $\frac{1}{2}$ gal.	=	qt.	$\frac{3}{8}$ gal. × 2	=	gal.
$\frac{1}{2}$ of $\frac{1}{2}$ gal.	=	gal.	$\frac{2}{8}$ gal. × 2	=	gal.
$\frac{1}{2}$ of $\frac{1}{4}$ gal.	=	pt.	$\frac{1}{4}$ gal. × 2	=	gal.
$\frac{1}{2}$ of $\frac{1}{4}$ gal.	=	gal.	$\frac{1}{8}$ gal. × 2	=	gal.
$\frac{1}{2}$ of $\frac{4}{8}$ gal.	=	qt.	$\frac{3}{4}$ gal. × 2	=	gal.
$\frac{1}{2}$ of $\frac{4}{8}$ gal.	=	gal.	$1\frac{1}{4}$ gal. × 2	=	gal.
$\frac{1}{2}$ of $\frac{2}{8}$ gal.	=	gal.	$2\frac{1}{2}$ gal. × 2	=	gal.
$\frac{1}{2}$ of $\frac{3}{4}$ gal.	=	pt.	$\frac{1}{2}$ gal. × 2	=	gal.
$\frac{1}{2}$ of $\frac{3}{4}$ gal.	=	gal.	$\frac{1}{2}$ gal. × 3	=	gal.

LESSON 64—CLASS WORK.

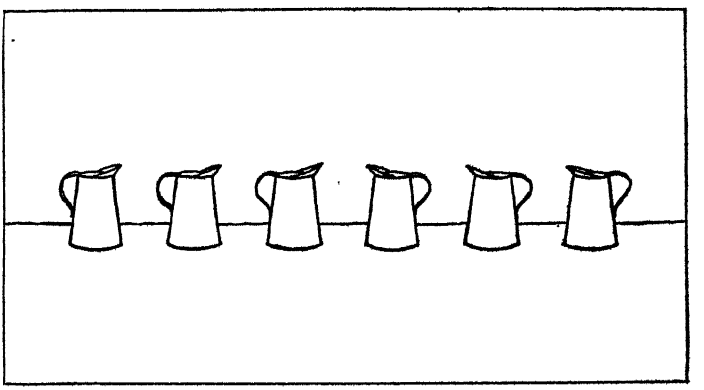
Place on the table the gallon measure. Put in it three quarts of water (level full). Does this make more or less than a gallon of water? What part of a gallon is it? Fill as many pint measures as you can with this $\frac{3}{4}$ of a gallon

of water. Place them in a row (grouping them by twos):



How many pint measures does it fill? How much water do the six measures contain? This pint measure has how much of a gallon in it? If one pint measure holds $\frac{1}{8}$ of a gallon, three pint measures will hold how much? Five pint measures? Six pint measures? (Ans. $\frac{6}{8}$ of a gallon.) Another name for $\frac{6}{8}$ of a gallon.

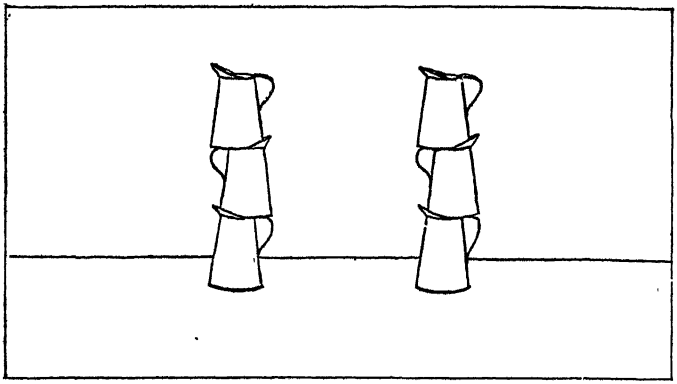
Place them in a row to show how many eighths.



How many eighths of a gallon in $\frac{3}{4}$ of a gallon?

$\frac{3}{4} \div \frac{1}{8} = ?$

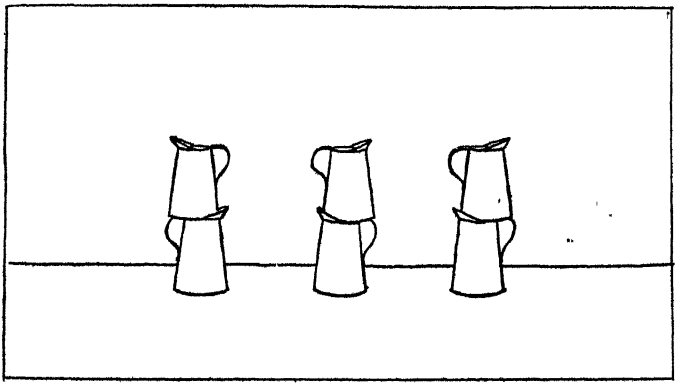
Place them in vertical rows to show how many $\frac{3}{8}$ the $\frac{3}{4}$ or $\frac{6}{8}$ will make.



(Statement: It will make two $\frac{3}{8}$.)

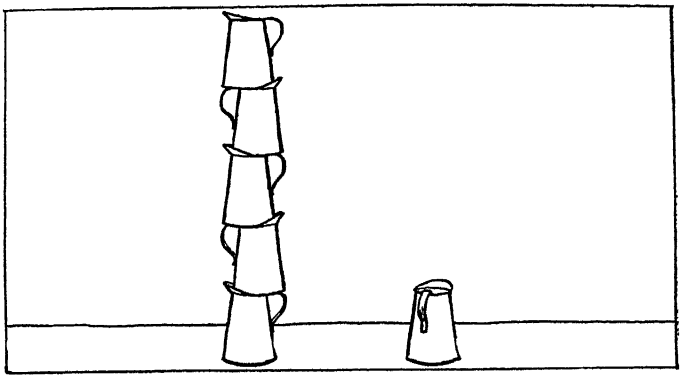
$$\frac{3}{4} \div \frac{3}{8} = ?$$

How many pints of water in a quart of water? A quart of water is what part of a gallon of water? Place them in vertical rows to show how many $\frac{1}{4}$'s in $\frac{3}{4}$ of a gallon:



$$\frac{3}{4} \div \frac{1}{4} = ?$$

Place them in vertical rows to show how many $\frac{5}{8}$-gallon in $\frac{3}{4}$-gallon.



(Statement: There are $1\frac{1}{3}$ $\frac{5}{8}$-gallon in $\frac{3}{4}$-gallon.)

$\frac{3}{4} \div \frac{5}{8} = ?$

DESK WORK.

$\frac{3}{4}$ gal. $\div$ $\frac{1}{4}$ gal. $= ?$	$\frac{1}{2}$ gal. $\div$ $\frac{3}{8}$ gal. $= ?$
$\frac{3}{4}$ gal. $\div$ $\frac{1}{8}$ gal. $= ?$	$\frac{1}{4}$ gal. $\div$ $\frac{1}{8}$ gal. $= ?$
$\frac{3}{4}$ gal. $\div$ $\frac{3}{8}$ gal. $= ?$	$\frac{1}{8}$ gal. $\div$ $\frac{1}{8}$ gal. $= ?$
$\frac{3}{4}$ gal. $\div$ $\frac{5}{8}$ gal. $= ?$	1 gal. $\div$ $\frac{1}{8}$ gal. $= ?$
$\frac{1}{2}$ gal. $\div$ $\frac{1}{4}$ gal. $= ?$	1 gal. $\div$ $\frac{3}{8}$ gal. $= ?$
$\frac{1}{2}$ gal. $\div$ $\frac{1}{8}$ gal. $= ?$	$\frac{5}{8}$ gal. $\div$ $\frac{3}{4}$ gal. $= ?$

LESSON 65—CLASS WORK.

Show me $\frac{1}{8}$ of a gallon of water.

Show me $\frac{2}{8}$ of a gallon of water.

Show me $\frac{1}{2}$ of $\frac{2}{8}$ of a gallon of water.

How many $\frac{1}{8}$'s of a gallon in $\frac{2}{8}$ of a gallon of water?

Show me $\frac{3}{8}$ of a gallon of water. Show me $\frac{1}{3}$ of $\frac{3}{8}$ of a gallon of water.

How many $\frac{1}{8}$'s of a gallon in $\frac{3}{8}$ of a gallon?

How many $\frac{2}{8}$ of a gallon in $\frac{3}{8}$ of a gallon?

How many $\frac{1}{8}$ of a gallon in $\frac{3}{8}$ of a gallon?
Show me $\frac{4}{8}$ of a gallon of water.
How many $\frac{1}{8}$'s of a gallon in $\frac{4}{8}$ of a gallon of water?
How many $\frac{2}{8}$ of a gallon in $\frac{4}{8}$ of a gallon of water?
How many $\frac{3}{8}$ of a gallon in $\frac{1}{2}$ of a gallon of water?
Show me $\frac{5}{8}$ of a gallon of water.
Group them to show how many $\frac{2}{8}$ in $\frac{5}{8}$ of a gallon.
Group them to show how many $\frac{3}{8}$ in $\frac{5}{8}$ of a gallon.
Group them to show how many $\frac{4}{8}$ in $\frac{5}{8}$ of a gallon.
Group them to show how many $\frac{1}{2}$-gallons in $\frac{5}{8}$ of a gallon.
Show me $\frac{7}{8}$ of a gallon of water.
Group them to show how many $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$ in $\frac{7}{8}$ of a gallon.
$\frac{1}{8}$ of a gallon is how many $\frac{2}{8}$ of a gallon?
$\frac{1}{8}$ of a gallon is how many $\frac{1}{4}$-gallons?
$\frac{1}{8}$ of a gallon is how many $\frac{3}{8}$ of a gallon?
$\frac{1}{8}$ of a gallon is how many $\frac{1}{2}$-gallons?
$\frac{1}{8}$ of a gallon is how many $\frac{5}{8}$ of a gallon?
$\frac{2}{8}$ of a gallon are how many $\frac{5}{8}$ of a gallon?
$\frac{3}{8}$ of a gallon are how many $\frac{5}{8}$ of a gallon?
$\frac{4}{8}$ of a gallon are how many $\frac{5}{8}$ of a gallon?
$\frac{4}{8}$ of a gallon are how many $\frac{6}{8}$ of a gallon?
$\frac{1}{2}$ of a gallon is how many $\frac{3}{4}$ of a gallon?
$\frac{5}{8}$ of a gallon are how many $\frac{3}{4}$ of a gallon?
$\frac{5}{8}$ of a gallon are how many $\frac{7}{8}$ of a gallon?
$\frac{3}{4}$ of a gallon are how many $\frac{7}{8}$ of a gallon?

Desk Work.

$\frac{1}{8}$ gal. $\div$ $\frac{1}{4}$-gal. = ?
$\frac{1}{8}$ gal. $\div$ $\frac{3}{8}$-gal. = ?
$\frac{3}{8}$ gal. $\div$ $\frac{5}{8}$-gal. = ?
$\frac{3}{8}$ gal. $\div$ $\frac{3}{4}$-gal. = ?
$\frac{3}{8}$ gal. $\div$ $\frac{7}{8}$-gal. = ?
$\frac{5}{8}$ gal. $\div$ $\frac{3}{4}$-gal. = ?
$\frac{5}{8}$ gal. $\div$ $\frac{7}{8}$-gal. = ?

$\frac{5}{8}$ gal. $\div$ $\frac{1}{2}$-gal. = ?
$\frac{1}{2}$ gal. $\div$ $\frac{5}{8}$-gal. = ?
$\frac{3}{4}$ gal. $\div$ $\frac{1}{2}$-gal. = ?
$\frac{3}{4}$ gal. $\div$ $\frac{3}{8}$-gal. = ?
$\frac{1}{4}$ gal. $\div$ $\frac{1}{8}$-gal. = ?
$\frac{3}{8}$ gal. $\div$ $\frac{1}{4}$-gal. = ?
$\frac{7}{8}$ gal. $\div$ $\frac{5}{8}$-gal. = ?

NOTE.—Place such work as the following on the board for drill.

$1\frac{1}{4} = \frac{}{4}$	$1\frac{3}{7} = \frac{}{7}$	$\frac{6}{8} = \frac{}{4}$
$2\frac{1}{4} = \frac{}{4}$	$1\frac{3}{4} = \frac{}{4}$	$\frac{6}{2} =$
$1\frac{1}{2} = \frac{}{2}$	$2\frac{1}{4} = \frac{}{4}$	$1 = \frac{}{3}$
$2\frac{1}{2} = \frac{}{2}$	$2 = \frac{}{3}$	$3 = \frac{}{3}$
$2\frac{1}{3} = \frac{}{3}$	$3\frac{1}{2} = \frac{}{2}$	$\frac{1}{3} = \frac{}{6}$
$2\frac{2}{3} = \frac{}{3}$	$1 = \frac{}{6}$	$\frac{2}{3} = \frac{}{6}$
$1\frac{1}{8} = \frac{}{8}$	$1\frac{1}{6} = \frac{}{6}$	$1\frac{2}{5} = \frac{}{5}$
$1\frac{1}{3} = \frac{}{3}$	$1\frac{1}{2} = \frac{}{6}$	$1\frac{1}{5} = \frac{}{5}$
$1\frac{2}{3} = \frac{}{3}$	$\frac{4}{8} = \frac{}{2}$	$1\frac{1}{5} = \frac{}{5}$
$1\frac{1}{7} = \frac{}{7}$	$\frac{4}{8} = \frac{}{4}$	$1\frac{3}{5} = \frac{}{5}$
$\frac{8}{2} =$	$\frac{3}{4}$ of $8 =$	$1\frac{3}{5} = \frac{}{5}$
$\frac{9}{3} =$	$\frac{2}{3} + \frac{1}{6} =$	$1\frac{2}{7} = \frac{}{7}$
$\frac{6}{3} =$	$2\frac{1}{2} \times 3 =$	$9 \div 4 =$
$\frac{8}{7} =$	$9 \div 2 =$	$9 \div 5 =$
$\frac{8}{3} =$	$5 \div 2 =$	$1\frac{3}{4} + \frac{1}{2} =$
$\frac{1}{6} \times 2 =$	$9 \div 3 =$	$1 \div \frac{1}{7} =$
$\frac{2}{3} \div \frac{1}{3} =$	$\frac{3}{4} - \frac{3}{8} =$	$\frac{1}{2}$ of $8 =$
$\frac{1}{3} \div \frac{1}{6} =$	$\frac{5}{8} - \frac{1}{4} =$	$\frac{1}{2}$ of $9 =$
$\frac{1}{2}$ of $\frac{1}{4} =$	$\frac{5}{8} \div \frac{1}{2} =$	$\frac{1}{2}$ of $7 =$
$\frac{2}{3}$ of $6 =$	$\frac{1}{2} \div \frac{3}{4} =$	$\frac{1}{2}$ of $6 =$

PRACTICAL PROBLEMS.

1. I made 2 banners out of 3 yards of satin; how long was each banner?

2. My sash was 5 yards long. I cut off $2\frac{1}{2}$ yards; how much was left?

3. A pole 8 feet long was broken in a storm, so that only 3 feet were left; how long was the part broken off?

4. I bought 7 yards of carpet for a stairway and used only $5\frac{1}{2}$ yards; how many yards were left?

5. Fred had a fishing line 2 yards long. He added 3 yards to it. How long was the line then?

6. A lady got 8 yards of muslin for window curtains. If she used 2 yards for each window, how many windows were there?

7. John has a kite string 8 yards long. If he winds up $3\frac{1}{2}$ yards of it, how far can the kite go?

8. A merchant tied 3 packages, using 2 yards of twine for each package; how much twine did he use?

9. May has a ribbon $3\frac{1}{2}$ feet long; Kate has one 2 feet long, and Nellie has a ribbon $2\frac{1}{2}$ feet long; how many yards have they together?

10. Mary bought 6 yards of silk for a dress, but found that she needed $7\frac{1}{4}$ yards; how much more did she buy?

---

1. A milkman had 3 quarts of milk and sold half of it; how many pints had he left?

2. If vinegar is worth 8 cents a gallon, how much will three quarts cost?

3. James has 7-eighths of a gallon of honey and Mary has 3-eighths of a gallon; how much more has James than Mary?

4. There are 5-eighths of a gallon of molasses in one jar and 1-fourth of a gallon in another jar; how many pints in both?

5. There was 1 gallon of water in a bucket and 3 quarts leaked out; how many pints were left in the bucket?

6. When milk is worth 2 cents a pint how much will 3-eighths of a gallon cost?

7. Sarah's mother had 1 quart of syrup, she used 1-half of it, then bought 3 pints more; how much did she then have?

8. 1-fourth of a gallon of water and 3-eighths of a gallon of water make how many gallons of water?

9. In 7-eighths of a gallon of water there are how many 1-half gallons of water?

10. Five-eighths of a gallon will make how many 1-fourth gallons?

11. If a quart of lemonade is worth 4 cents, how much are 3 pints worth?

12. If 3 pints of chocolate cost 6 cents, how much will 1-half of a gallon cost?

13. A lady had a gallon of vinegar, she used 1 pint each day; how many weeks did it last her?

14. Cider is worth 4 cents a quart and vinegar is worth 6 cents a quart; how much more will a quart of vinegar cost than a quart of cider?

---

1. John's mother bought 3 quarts of cranberries one day and 1 quart the next day; how many pecks did she buy?

2. James picked 3 pints of beans; his mother cooked a quart. How many pints were left?

3. A grocer has a bushel basket of potatoes; a lady buys 1 peck. How much of a bushel is left?

4. A grocer has a bushel basket of apples; a lady buys $\frac{1}{2}$ peck. How many ladies can get a half peck from this basket?

5. Mary's mother buys $\frac{1}{2}$ bushel of tomatoes and Tom's mother buys 1 peck. How many pecks were bought?

6. The grocer has a peck of peaches and sells 6 quarts. How much of a peck did he sell?

7. John has 7 quarts of cherries and gives Charley 5 quarts; how much of a peck has he left?

8. Mary has 5 pints of cherries and gives Jennie $\frac{1}{2}$ of them; how many quarts and pints did she give Jennie?

9. John has 2 quarts of cherries, Mary 3 quarts and Susie 3 quarts. How much of a bushel have all together?

10. If you buy $\frac{1}{8}$ of a peck of apples how many quarts would you get?

CHAPTER VIII.

Ten—Eleven—Ninths—Tenths.

Lesson 65—Class Work.

Note.—Introduce the study of *ten* through the 1-inch unit See Lessons on Six and Seven

Place three yard measures end to end. How long is the line of measures? I will add another measure one foot long; how long is the line now? Nine feet and how many feet make ten feet? Nine feet are how many yards? In ten feet there are how many yards?

Note.—Use the measures adapting the illustration to the question. Refer to previous lessons in which long measure is employed

In six feet how many yards?
In seven feet how many yards?
In eight feet how many yards?
In five feet how many yards?
In four feet how many yards?
In three feet how many yards?
In one foot?
In two feet?
In ten feet?

Note.—Review:—

$\frac{1}{3}$-yard is how many feet?
1 yard is how many feet?
$\frac{2}{3}$-yard are how many feet?
$1\frac{1}{3}$ yards are how many feet?
2 yards are how many feet?

$1\frac{2}{3}$ yards are how many feet?
3 yards are how many feet?
$2\frac{2}{3}$ yards are how many feet?
$2\frac{1}{3}$ yards are how many feet?
$3\frac{1}{3}$ yards are how many feet?

DESK WORK.

3 yd. + 1 ft. = ft.	10 ft. − $1\frac{1}{3}$ yd. = yd.
3 ft. + 3 ft. + 3 ft. = ft.	10 ft. − 2 yd. = ft.
2 yd. + 4 ft. = ft.	10 ft. ÷ 3 ft. =
5 ft. + 5 ft. = ft.	10 ft. ÷ 5 ft. =
5 ft. + 5 ft. = yd.	5 ft. × 2 = yd. ft.
$2\frac{2}{3}$ yd. + 2 ft. = ft.	5 ft. × 2 = yd.

LESSON 67—CLASS WORK.

Place a gallon of water on the table.

How many pints in the gallon? How many pints must be added to the eight pints to make ten pints? Place two pints of water on the table. Eight pints and how many pints make ten pints? Ten pints are how many gallons? Take eight pints from ten pints; how many gallons are left? In ten pints there are how many eight-pints? Or, how many 8's of pints?

NOTE.—Review:—

Ten pints are how many gallons?
Eight pints are how many gallons?
Nine pints are how many gallons?
Four pints are how many gallons?
Six pints are how many gallons?
Seven pints are how many gallons?
Five pints are how many gallons?
Three pints are how many gallons?
One pint is how many gallons?
Two pints are how many gallons?
In ten pints there are how many eight-pints?

In eight pints there are how many eight-pints?
In nine pints there are how many eight-pints?
In six pints there are how many eight-pints?
In five pints there are how many eight-pints?
In seven pints there are how many eight-pints?
In four pints there are how many eight-pints?
In two pints there are how many eight-pints?
In three pints there are how many eight-pints?
In one pint there is how many eight-pints?
One gallon is how many pints?
One and one fourth gallons are how many pints?
One and one eighth gallons are how many pints?
One half gallon is how many pints?
Three-fourths gallon are how many pints?

DESK WORK.

8 pt. + pt. = 10 pt.
10 pt. − 1 pt. = gal.
10 pt. − 8 pt. = gal.
10 pt. − 2 pt. = gal.
$1\frac{1}{4}$ gal. − 5 pt. = gal.
10 pt. ÷ 8 pt. =

10 pt. ÷ 5 pt. =
$\frac{1}{2}$ of 10 ft. = yd.
$\frac{1}{5}$ of 10 pt. = gal.
$1\frac{1}{8}$ gal. − 1 pt. = gal.
$\frac{3}{4}$ gal. + $\frac{1}{2}$ gal. = gal. pt.
$\frac{3}{4}$ gal. + $\frac{1}{2}$ gal. = gal.

LESSON 68—CLASS WORK.

Place on the table a peck of wheat.

How many quarts are there?

How many quarts must be added to the 8 quarts to make 10 quarts?

Place on the table 2 quarts of wheat. 8 quarts of wheat and 2 quarts of wheat are how many quarts of wheat? 1 quart of wheat is what part of a peck of wheat? 2 quarts are what part of a peck? 10 quarts are how many pecks of wheat? How many 2's of quarts in 8 quarts (or a peck)? How many 2's of quarts in 10 quarts of wheat?

Show me a 2 of quarts of wheat. Show another 2 of quarts. Another. Another. How many have you shown? How many more 2's would make the 10 quarts of wheat?

What are 5 times 2 quarts of wheat in quarts? In pecks?

8 quarts are how many pecks?
9 quarts are how many pecks?
10 quarts are how many pecks?
1 quart is how many pecks?
3 quarts are how many pecks?
4 quarts are how many pecks?
2 quarts are how many pecks?
6 quarts are how many pecks?
5 quarts are how many pecks?
7 quarts are how many pecks?
$\frac{1}{2}$-peck is how many quarts?
1 peck is how many quarts?
$\frac{3}{4}$-peck are how many quarts?
$\frac{1}{4}$-peck is how many quarts?
$\frac{5}{8}$-peck are how many quarts?
$\frac{3}{8}$-peck are how many quarts?

NOTE.—Develop the 5's in 10; also, 4's in 10.

DESK WORK.

8 qt. + 2 qt. = qt.
8 qt. + 2 qt. = pk.
6 qt. + 4 qt. = pk. qt.
10 qt. − 3 qt. = pk.
10 qt. ÷ 5 qt. =
10 qt. ÷ 4 qt. =
10 qt. ÷ 2 qt. =

10 qt. ÷ 3 qt. =
$1\frac{1}{2}$ pk. + 1 qt. = qt.
$\frac{1}{2}$ pk. + $\frac{3}{4}$ pk. = qt.
$\frac{1}{2}$ pk. + $\frac{3}{4}$ pk. = pk.
3 qt. × 3 = pk. qt.
3 qt. × 3 = pk.
$\frac{1}{2}$ of 10 qt. = pk.

LESSON 69—CLASS WORK.

Here is a dime; who can tell me how many one-cent pieces are equal to a dime?

Place them in a row on the table.

How many in the row? What single piece of money are they all equal to? Which would you rather have, a dime or ten 1-cent pieces? Why? Which would buy the more? Now, place them in two rows with the same number in each. How many in this row? How many in this? What single piece of money is this row equal to? What single piece is this row equal to? Put a nickel in front of this row; now one in front of this one. What single piece of money would you be willing to give for the two nickels? How many nickels make a dime? One half of a dime is how many nickels? One half of a dime is how many cents? How many 5's of 1-cent pieces make a dime? One nickel and one nickel make what? Five 1-cent pieces and five 1-cent pieces are what? How many 5's of cents in ten cents? Place the 1-cent pieces in rows with just two in a row? How many rows? How many 2's of cents in ten cents? How many 2's of cents in a dime? Five times two cents are how many cents? Five times two cents are how *much* money? Place them again in two equal rows. I will take one cent from this row; recite, John. (Statement: 1 cent taken from 5 cents leaves 4 cents.) I will now put it in the other row; Mary, recite. (Statement: 5 cents and 1 cent are 6 cents.) How much in both rows? (Ans. In both rows there are 6 cents and 4 cents, which are 10 cents.) Make the rows equal again.

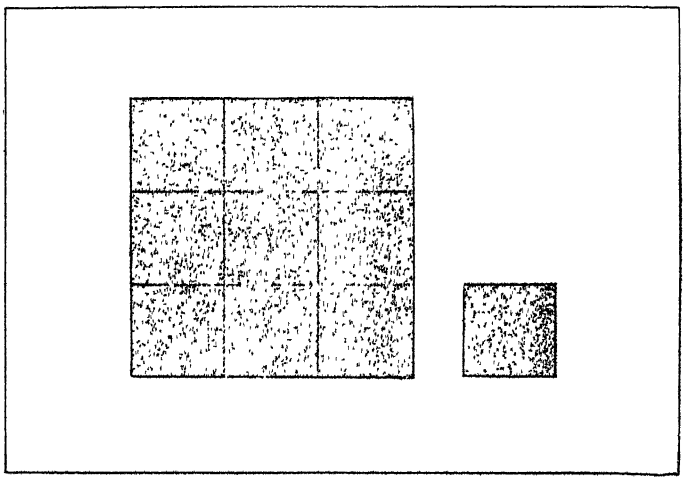
NOTE.—Repeat, taking different numbers of cents each time from one row and putting them in the other, calling on different pupils to recite. In this way develop the parts of ten. As a review ask such questions as: What two numbers of cents make 10 cents? What other two numbers of cents make 10 cents? etc.

DESK WORK.

10 ct. ÷ 5 ct. =	8 ct. ÷ 2 ct. =
5 ct. × 2 = ct.	6 ct. + 4 ct. = ct.
10 ct. ÷ 2 ct. =	7 ct. + 3 ct. = ct.
2 ct. × 5 = ct.	8 ct. + 2 ct. = ct.
5 ct. − 2 ct. = ct.	5 ct. + 4 ct. = ct.

5 ct. + 2 ct. = ct.	$\frac{1}{2}$ of 10 ct. = ct.
5 ct. − 3 ct. = ct.	$\frac{1}{2}$ of 8 ct. = ct.
5 ct. + 3 ct. = ct.	$\frac{1}{2}$ of 6 ct. = ct.
5 ct. − $\frac{1}{4}$ ct. = ct.	$\frac{3}{4}$ of 8 ct. = ct.
5 ct. + 4 ct. = ct.	$\frac{2}{3}$ of 6 ct. = ct.
10 ct. − 3 ct. = ct.	$\frac{2}{3}$ of 9 ct. = ct.
10 ct. − 1 ct. = ct.	$\frac{1}{2}$ of 10 ct. = ct.
9 ct. ÷ 3 ct. =	$\frac{1}{2}$ of 5 ct. = ct.
8 ct. ÷ 4 ct. =	$\frac{1}{2}$ of $2\frac{1}{2}$ ct. = ct.

NOTE.—Introduce the study of *eleven* through the use of the money units, employing the dime and one cent, two nickels and one cent, and eleven cents. Devote one lesson to eleven, using linear measure, one, using liquid measure, and one, using dry measure. Model the lessons after those given on other numbers and follow each with appropriate desk work.



LESSON 70—CLASS WORK.

NOTE.—Draw on the blackboard a square yard and a square foot in separate diagrams, shading both as in the figures. Impart some idea of surface. Show that with the square foot surface may be measured. Measure the square yard of surface with a pasteboard measure a foot long and a foot wide.

What is surface? Can surface be measured? What is a square foot? What is a square yard? How many square feet in a square yard? Count the square feet by *threes* from top to bottom as I point them out. Now from right to left.

What is one-third of a square yard? What are two-thirds of a square yard? What are three-thirds of a square yard? How many square feet in a square yard? One square foot is what part of a square yard? Two square feet? Four square feet? Five square feet? Seven square feet? Eight square feet? Nine square feet?

Three square feet are what part of a square yard? What other name for it? Three times one-ninth of a square yard are what? One-third of one-third of a square yard is what? How many one-ninths of a square yard in one-third of a square yard?

Six feet are what part of a square yard? What other name for it? Which is greater, two-thirds of a square yard or six-ninths of a square yard?

DESK WORK.

3 sq. ft. + 3 sq. ft. + 3 sq. ft. = sq. ft.
6 sq. ft. + 3 sq. ft. = sq. ft.
9 sq. ft. − 4 sq. ft. = sq. ft.
5 sq. ft. + 4 sq. ft. = sq. ft.
9 sq. ft. − 3 sq. ft. = sq. ft.
5 sq. ft. + 5 sq. ft. = sq. ft.
11 sq. ft. − 2 sq. ft. = sq. ft.
5 sq. ft. + 6 sq. ft. = sq. ft.
11 sq. ft. − 4 sq. ft. = sq. ft.
8 sq. ft. + 3 sq. ft. = sq. ft.
10 sq. ft. ÷ 3 sq. ft. =
10 sq. ft. ÷ 2 sq. ft. =
10 sq. ft. ÷ 4 sq. ft. =
9 sq. ft. ÷ 3 sq. ft. =
9 sq. ft. ÷ 6 sq. ft. =
10 sq. ft. ÷ 7 sq. ft. =

$\frac{1}{3}$ sq. yd. $-$ $\frac{1}{9}$ sq. yd. = sq. yd.
$\frac{1}{3}$ sq. yd. $+$ $\frac{1}{9}$ sq. yd. = sq. yd.
$\frac{1}{3}$ sq. yd. $\div$ $\frac{1}{9}$ sq. yd. =
$\frac{2}{3}$ sq. yd. $\div$ $\frac{1}{9}$ sq. yd. =
$\frac{1}{2}$ of $\frac{2}{3}$ sq. yd. = sq. yd.
$\frac{1}{3}$ of $\frac{1}{3}$ sq. yd. = sq. yd.
$\frac{1}{3}$ of $\frac{2}{3}$ sq. yd. = sq. yd.
$\frac{2}{3}$ sq. yd. $+$ $\frac{1}{9}$ sq. yd. = sq. yd
$\frac{2}{3}$ sq. yd. $-$ $\frac{1}{9}$ sq. yd. = sq. yd.
$\frac{7}{9}$ sq. yd. $-$ $\frac{1}{3}$ sq. yd. = sq. yd.

Lesson 71—Desk Work.

Place on the table one dime.

Put in a row as many 1-cent pieces as will make the row worth as much as the dime. How many will it take, class? If I take up one of the row, how much of the whole row do I take up? One cent is how much of ten cents? How much of a dime? Put a nickel on the table and put five 1-cent pieces in a row. Which would you rather have? One cent is how much of 5 cents? How much of a nickel is it? If one cent is one fifth of a nickel what is one fifth of two nickels? Then, what is one fifth of a dime? Group the ten 1-cent pieces to show the *fifths* in a dime. How many *fifths* does it make? Show me *again* one *tenth* of a dime. Show me *again* one *fifth* of a dime. One tenth of a dime is how much of one fifth of a dime? How many one tenths of a dime in one fifth of a dime? If one cent is one tenth of a dime three cents are ——. If one cent is one tenth of a dime two cents are ——. What other part of a dime? Four cents are what part of a dime? What other part? Five cents are what part of a dime? What *other part*? Six cents are what part of a dime? What *other* part? Seven cents are what part of a dime? Eight cents? What *other* part? Nine cents are what part of a dime? Ten cents?

How many cents in one fifth of a dime?
How many cents in two fifths of a dime?
How many cents in three fifths of a dime?
How many cents in four fifths of a dime?
How many cents in five fifths of a dime?
How many cents in one tenth of a dime?
How many cents in two tenths of a dime?
How many cents in three tenths of a dime?
How many cents in four tenths of a dime?
How many cents in seven tenths of a dime?
How many cents in five tenths of a dime?
How many cents in one half of a dime?
How many cents in six tenths of a dime?
How many cents in eight tenths of a dime?
How many cents in nine tenths of a dime?
One fifth of a dime is how many tenths of a dime?
Two fifths of a dime are how many tenths of a dime?
Three fifths of a dime are how many tenths of a dime?
Four fifths of a dime are how many tenths of a dime?
Five fifths of a dime are how many tenths of a dime?

Desk Work.

Note.—Continue class work with objects until pupils are well prepared for the following:

1 dime $\div \frac{1}{10}$ dimes =
1 dime $\div \frac{1}{5}$ dimes =
$\frac{1}{5}$ d. $\div \frac{1}{10}$ d. =
$\frac{1}{10}$ d. $\div \frac{1}{5}$ d. =
$\frac{1}{10}$ d. $\times 2$ = d.
$\frac{1}{2}$ of $\frac{1}{5}$ d. = d.
$\frac{3}{10}$ d. $\div \frac{1}{5}$ d. =
$\frac{1}{5}$ d. $\div \frac{3}{10}$ d. =
$\frac{1}{5}$ d. $+ \frac{1}{10}$ d. = d.
$\frac{2}{5}$ d. $+ \frac{3}{10}$ d. = d.
$\frac{4}{5}$ d. $\div \frac{1}{10}$ d. =
$\frac{1}{2}$ d. $\div \frac{1}{10}$ d. =

$\frac{1}{2}$ d. $\div \frac{1}{5}$ d. =
$\frac{1}{10}$ d. $\div \frac{1}{2}$ d. =
$\frac{1}{5}$ d. $\div \frac{1}{2}$ d. =
$\frac{1}{2}$ d. $\div \frac{3}{5}$ d. =
$\frac{4}{5}$ d. $\div \frac{3}{5}$ d. =
$\frac{3}{5}$ d. $\div \frac{4}{5}$ d. =
$\frac{1}{2}$ d. $+ \frac{3}{10}$ d. = d.
$\frac{1}{2}$ d. $+ \frac{1}{5}$ d. = d.
$\frac{7}{10}$ d. $+ \frac{1}{5}$ d. = d.
$\frac{7}{10}$ d. $- \frac{1}{5}$ d. = d.
1 d. $\div \frac{1}{2}$ d. =
$\frac{4}{5}$ d. $- \frac{1}{2}$ d. =

PART SECOND.

CHAPTER IX.

Suggestions—Signs—Twelve.

OUTLINE FOR SECOND YEAR.

NOTE.—Devote as many lessons to reviews of first grade work as may be necessary.

First month, teach twelve.

Second month, teach fourteen, fifteen, sixteen, and seventeen.

Third month, teach eighteen, nineteen, and twenty.

Fourth month, teach to twenty-five inclusive.

Fifth month, teach to thirty inclusive.

Sixth month, teach to thirty-seven.

Seventh month, teach to forty-four.

Eighth month, teach to fifty inclusive.

Ninth month, general review.

SUGGESTIONS.

1. Let the "class" work be objective. *Frequent* drills for facility and accuracy should *follow*.

2. Let the *objects* used be those for the *measurement* of which there is a mental and industrial necessity, and not those commonly used; as, toothpicks, splints, shoe pegs, pebbles, apples, marks, etc., to which no such interest attaches.

3. Do not fear an *excessive use* of right objects; *the danger lies in the other direction.*

4. Use definitely defined units; as, the foot rather than a circle, or the pound rather than simply a square.

5. As the quantitative idea and not the qualitative one is important, any material may be used to represent any other. For instance, when water is measured, it may be called milk, or vinegar, or syrup; when salt is weighed, it may be called sugar, or starch, or rice, etc.

6. The rule should be, in expressing number ideas illustrated by objects, to require pupils to include the *names of objects* used in illustrations, in their explanations, with the exception mentioned in suggestion 5.

7. Let each number be carried through three stages—the development stage, the use stage, and the drill stage—in the order in which these steps are named.

8. Let explanations of pupils be as brief and clear as possible.

9. Aim at accuracy and correctness of expression as well as of thought.

10. Some units of measure are well adapted to the developement of one series of fractions and some, another; as, the *gallon*, halves, fourths, and eighths; the *pound*, halves, fourths, eighths, and sixteenths; the *foot*, halves, thirds, fourths, sixths, and twelfths; the *dime*, halves, fifths, and tenths; the *yard*, thirds, ninths, eighteenths, and thirty-sixths. Reference to the lessons that follow will make this clear. There is plenty of this kind of work for practice in the first and second grades, so that pupils in third and fourth grades, with the least direction from the teacher, can deal in an abstract way, with fractions that cannot be referred to a common unit of measure.

11. The lessons given in Part Second are based on the numbers between ten and twenty-one but it has been demonstrated that the best classes can accomplish the work to *fifty*, and the poorer ones anywhere from *thirty* to *forty*. See "Outline for Second Year".

12. Give a variety of work in each lesson as a rule. Do not neglect any *kind of work* for a long time, if you wish to get the best results.

THE SIGNS.

Read:—

+ *and.*

− *taken from* or *from*, beginning with the number on its right.

= *are*, with +.

= *leaves*, with −.

× *times*, beginning with the number on its right.

÷ *how many* (3's) *in* (6), in connection with =, beginning with the number on its right. Or,

÷ (6) *is how many times* (3), beginning with the number on its left.

LESSON 1—CLASS WORK.

NOTE.—Each pupil must have a foot measure and twelve one-inch sticks.

How long is one of the little sticks that I have given you, children? Are they all the same length? As they are all just one inch long, we shall call them one-inch measures. You may each hold up a one-inch measure. Now, let us find how many one-inch measures it will take to make a line just as long as the foot measure. (Pupils put the measures in a line and ascertain.) How many does it take? In one foot there are how many inches? Now, let us make two lines of the twelve one-inch measures. Every time you place a stick in one line place one in the other. How many inch measures in each line? Then, how long is each line? Test the lines with the foot measure.

NOTE.—A six-inch measure would be better for this.

Six inches and six inches are how many inches? How many six-inch lines are on your desk? Two times six inches are how many inches? In twelve inches there are how many six-inches? How many six-inch lines did you

make with the twelve inch-measures? Then, each line is what part of twelve inches? One half of twelve inches is how many inches?

This time make three lines of equal length of the twelve one-inch measures. Find how many measures in each line. How many lines have you made? How long is each line? How many fours of inches in twelve inches? In a foot? Hold up the foot measure and show me the *three* fours of inches in a foot. Four inches and four inches and four inches are how many inches? Place your hand on one four-inch line. On two four-inch lines. Two times four inches are how many inches? Place your hand on three times four-inch lines. Three times four inches are how many inches?

NOTE.—Four-inch measures may be here used to advantage.

Four inches are what part of a foot? Show me one third of a foot. Show me two thirds of a foot. Show me three thirds of a foot. How many inches in a foot? How many inches in one third of a foot? How many inches in two thirds of a foot? In three thirds of a foot?

DESK WORK.

1 ft. = in.	6 in. + 6 in. = ft.
$\frac{1}{2}$ ft. = in.	1 ft. − 4 in. = ft.
12 in. ÷ 6 in. =	12 in. − 8 in. = ft.
$\frac{1}{3}$ ft. = in.	1 ft. − 6 in. = ft.
1 ft. ÷ 4 in. =	1 ft. − 6 in. = in.
$\frac{2}{3}$ ft. = in.	$\frac{1}{2}$ ft. − $\frac{1}{3}$ ft. = in.

LESSON 2—CLASS WORK.

Now, make four lines of the twelve one-inch measures. How many inch measures did you place in each line? How long is each line?

NOTE.—Have pupils test the length of the lines by using the foot measure or by using a three-inch measure.

How many three-inch lines have you made? Then, how many *threes* of inches in twelve inches? Show me one *three* of inches on the foot measure. Now, show two *threes* of inches with your right hand and separate them with your left. How many *threes* of inches below your right hand? How many above? How many in all? How many *threes* of inches in a foot? What are two times three inches? Three inches and three inches are how many inches? Three inches, three inches, and three inches are how many inches? Three times *three* inches are how many inches? Nine inches and three inches are how many inches? Four times *three* inches are how many inches? How many *threes* of inches in six inches? In twelve inches? In a foot? In nine inches?

Now, make six lines of the twelve one-inch measures. How many inch measures in each line? How long is each line? How many *twos* of inch measures in twelve inch measures? How many *twos* of inches in twelve inches? How many *twos* of inches in a foot? Show me a *two* of inches on your foot measure. Two *twos*; three *twos*; four *twos*; five *twos*; six *twos*. Four *twos* are how many inches? Show me. Three *twos* are how many inches? Show me. Five *twos* are how many inches? Show me. Six *twos* are how many inches?

Make the two-inch lines again. How *many* lines are there? Then two inches are what part of twelve inches? Four inches are what part of a foot? What other part? Six inches? What other part or name? Eight inches? Ten inches? Twelve inches?

NOTE.—Review with many questions on the parts of a foot.

DESK WORK.

12 in. ÷ 3 in. =	3 in. × 4 = in.
12 in. ÷ 6 in. =	8 in. + 2 in. ft.
12 in. ÷ 4 in. =	12 in. - 8 in. = ft.
6 in. × 2 = in.	10 in. − 4 in. = ft.
4 in. × 3 = ft.	4 in. — 2 in. = ft.

NOTE.—Add other similar questions.

Lesson 3—Class Work.

Form a line, using twelve foot measures.

Note.—Let a sufficient number of pupils stand before the class to hold the measures up in a line end to end. Separate the line in the middle.

How long is this part? How long is this part? How many *sixes* of feet in twelve feet? What part of the whole line is this part? What part of the whole line is this part? One half of twelve feet are how many feet? Two halves of twelve feet are how many feet? Now break the parts three feet from this end; now three feet from this end. How long is the first part? How long is the second part? The third part? The fourth part? How many parts are there in the whole line? How many *threes* of feet in twelve feet? In six feet? In nine feet? How many yards in three feet? In six feet? In nine feet? In twelve feet?

Note.—The teacher may now substitute four yard measures for the twelve foot measures.

Is the line now just as long as it was at first? Separate the line into as many parts as there are yards. How many parts are there? How many yards are there? One yard is how much of the twelve feet? Two yards is how much of the twelve feet? Another name? Three yards are how much of the twelve feet? Four yards? What is one half of twelve feet in yards? One fourth? Three fourths?

Note.—Return to the twelve one-foot measures.

How long is this line? Now separate it four feet from this end. How long is this end? How long is the middle? This end? How many *fours* of feet in twelve feet? In eight feet? One part is what part of the whole line? Two parts? Three parts? What is one third of twelve feet? In yards? Two thirds of twelve feet? In yards? Three thirds of twelve feet? In yards?

Note.—Similarly show the *twos* of feet in twelve feet.

Desk Work.

6 ft. + 6 ft. = ft. = yd.	6 ft. + 4 ft. = yd.
6 ft. + 3 ft. = ft. = yd.	11 ft. = yd. ft.
6 ft. × 2 ft. = yd. = ft.	8 ft. = yd. ft.
12 ft. ÷ 2 ft. =	$\frac{1}{2}$ of 12 ft. = ft.
12 ft. ÷ 4 ft. =	$\frac{1}{3}$ of 12 ft. = ft.
12 ft. ÷ 3 ft. =	$\frac{1}{4}$ of 12 ft. = ft.
7 ft. + 5 ft. = yd.	$\frac{3}{4}$ of 12 ft. = ft.
6 ft. + 5 ft. = yd.	$\frac{5}{6}$ of 12 ft. = ft.

Lesson 4—Class Work.

Note.—Let three pupils stand before the class and, with four yard measures placed end to end, form a horizontal line. The teacher should stand back of the pupils with two foot measure *at hand*.

How many yards long is this line of measures? One yard is how many feet? Two yards are how many feet? Three yards? Four yards? How many feet long is the line? Then, in twelve feet, there are how many yards? In nine feet (teacher removing one yard measure) there are how many yards? (Answer form: In nine feet there are three yards.) In ten feet (teacher adding one foot measure) there are how many yards? In eleven feet (teacher adding another foot measure) there are how many yards? In eight feet (teacher seeing that the line is just eight feet long) there are how many yards? In six feet there are how many yards? In seven feet there are how many yards? In five feet how many yards? In four feet how many yards? In three feet? In two feet? In one foot?

Note.—This may be repeated two or three times with profit. In the hands of a skillful teacher this exercise is an excellent one, the illustrative feature entailing no loss of time whatever.

DESK WORK.

12 ft. = yd.	10 ft. = yd.	11 ft. = yd. ft.
6 ft. = yd.	4 ft. = yd.	11 ft. = yd.
7 ft. = yd.	3 ft. = yd.	9 ft. ÷ 3 ft. =
9 ft. = yd.	1 ft. = yd.	12 ft. ÷ 3 ft. =
8 ft. = yd.	2 ft. = yd.	7 ft. ÷ 3 ft. =
11 ft. = yd.	8 ft. = yd. ft.	5 ft. ÷ 3 ft. =
5 ft. = yd.	8 ft. = yd.	11 ft. ÷ 3 ft. =

LESSON 5—CLASS WORK.

NOTE.—Have on the table a peck measure and four quart measures. Dry measure is introduced in the first year's work, on the number *eight*, so pupils are ready for the work following. Use any convenient material, as corn, wheat, oats, etc.

How many quarts of corn in a peck of corn? How many *fours* of quarts in a peck of corn? A peck of corn and four quarts of corn are how many quarts of corn? How many *fours* of quarts of corn? Four quarts and four quarts and four quarts are how many quarts? How many pecks of corn? Three times four quarts of corn are how many quarts of corn? Two times four quarts of corn? How many *twos* of quarts in four quarts of corn?

NOTE.—Let this be shown by *grouping* the four quarts of corn in *twos* of quarts.

How many twos of quarts in this peck of corn? How many twos of quarts in the peck and this half-peck of corn? How many *twos* of quarts in the twelve quarts of corn? Two times two quarts of corn are how many quarts of corn? Four times two quarts of corn are how many quarts of corn? Six times two quarts of corn are how many quarts of corn?

NOTE.—Fill two more quart measures from the peck measure in use.

Two quarts of corn taken from a peck of corn leaves how many quarts of corn? Four quarts of corn and two quarts of corn are how many quarts of corn? How many

sixes of quarts are on the table? How many sixes of quarts in twelve quarts? Show me one half of twelve quarts. Show me two halves of twelve quarts. Two times six quarts of corn are how many quarts of corn? Six quarts and six quarts are how many quarts? Show (by grouping) how many *threes* of quarts in this six quarts of corn. How many *threes* of quarts now in this peck measure? Two *threes* of quarts and two *threes* of quarts are how many *threes* of quarts? Two times three quarts are how many quarts? Four times three quarts are how many quarts? Empty two quarts back into the peck measure. How many *eights* of quarts in twelve quarts of corn? How many pecks?

DESK WORK.

8 qt. + 4 qt. = qt.	$\frac{1}{2}$ of 12 qt. = qt.
12 qt. ÷ 4 qt. =	$\frac{1}{4}$ of 12 qt. = qt.
12 qt. ÷ 3 qt. =	$\frac{2}{3}$ of 12 qt. = qt.
12 qt. ÷ 6 qt. =	$\frac{1}{6}$ of 12 qt. qt.
12 qt. ÷ 2 qt. =	$\frac{3}{4}$ of 12 qt. = qt.
6 qt. + 6 qt. = qt.	$\frac{1}{6}$ of 12 qt. = qt.
12 qt. ÷ 8's of qt. =	$\frac{3}{8}$ of 12 qt. = qt.
4 qt. × 2 = qt.	$\frac{5}{8}$ of 12 qt. = qt.
4 qt. × 3 = qt.	$\frac{7}{8}$ of 12 qt. = qt.
3 qt. × 2 = qt.	4 qt. + 3 qt. + qt. = 12 qt.
3 qt. × 3 = qt.	10 qt. + qt. = 12 qt.

LESSON 6.—CLASS WORK.

How many inches in a foot? Who can tell me how many eggs in a dozen? How many eggs in a half dozen? What is one fourth of a dozen eggs? What are two fourths of a dozen eggs? What is the difference between two fourths of a dozen eggs and one half dozen eggs? What are three fourths of a dozen eggs? Four fourths? What is one third of a dozen eggs? Two thirds of a dozen

eggs? What are two thirds of twelve inches? Show me on your foot measures two thirds of a foot. Show me one sixth of a foot. Show me three sixths of a foot. Which is more, three sixths of a foot or one half of a foot? What is one sixth of a dozen eggs? Three sixths of a dozen eggs? What else are six eggs? What would a dozen eggs cost at one cent a piece? A half dozen eggs? One fourth of a dozen eggs? Three fourths of a dozen eggs? One third of a dozen would cost what? Two thirds of a dozen would cost what? What would a dozen peaches cost at one half cent a piece? What would a half dozen cost? One fourth of a dozen? At four for a nickel, how many nickels would pay for a dozen apples? At three for a nickel, how many nickels would pay for a dozen bananas?

DESK WORK.

$12 \div 6 =$	$8 + \quad = 12$
$6 \times 2 =$	$4 \times 3 =$
$12 \div 2 =$	$4 + 6 =$
$\frac{1}{2}$ of $12 =$	$3 \times 4 =$
$\frac{2}{3}$ of $12 =$	$4 + 4 + 4 =$
$\frac{3}{4}$ of $12 =$	$3 + 3 + 3 + \quad = 12$
$6 + 6 =$	$12 - 5 =$
$7 + \quad = 12$	$12 - 3 =$

LESSON 7—CLASS WORK.

NOTE.—Review all fractions so far taught. Simple analyses may now be introduced. When the fractions under any "Unit of Reference" have been carefully studied and the pupils are able to explain them satisfactorily, place them on the board *to remain* for occasional drill *without objects* and *without analysis.*

THE GALLON AS A UNIT OF REFERENCE.

NOTE.—Let the problem, $\frac{1}{4} + \frac{1}{4} =$ what? be written on the blackboard. The child, under direction of the teacher, fills a *quart* measure, using the pint measure to fill it; also fills a pint measure. At the proper time and place he *shows* the two quantities, as he proceeds with the solution, to illustrate what he is saying as he

gives the following analysis One fourth of a gallon of water is two pints, *one* pint is one eighth of a gallon of water, *two* pints are two eighths of a gallon of water, one eighth of a gallon of water and two eighths of a gallon of water (pouring both into the gallon measure) are three eighths of a gallon of water. Therefore $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$

The following is a list of problems to which this analysis can be readily adapted —

$\frac{1}{2} + \frac{3}{4} = ?$	$\frac{3}{4} - \frac{1}{2} = ?$	$\frac{1}{8} \times 2 = ?$
$\frac{1}{8} + \frac{1}{2} = ?$	$\frac{1}{2} - \frac{1}{8} = ?$	$\frac{3}{8} \times 2 = ?$
$\frac{1}{8} + \frac{1}{4} = ?$	$\frac{1}{2} \div \frac{1}{8} = ?$	$\frac{1}{2}$ of $\frac{1}{4} = ?$
$\frac{3}{4} + \frac{1}{8} = ?$	$\frac{3}{4} \div \frac{1}{2} = ?$	$\frac{1}{2}$ of $\frac{1}{2} = ?$
$\frac{3}{4} - \frac{1}{8} = ?$	$\frac{3}{4} \div \frac{1}{8} = ?$	$\frac{5}{8} - \frac{1}{2} = ?$

LESSON 8—CLASS WORK.

THE SQUARE YARD AS A UNIT OF REFERENCE.

NOTE.—The work under this head may be postponed until after surface measure has been introduced; but it is given here that those who wish to do so may use it; but little preparation is necessary to make it intelligible to the children.

1. They should have some notion of surface.

2. They should know that the square foot and square yard are but suitable *units* for the measurement of surface.

3. They should have a good idea of their *value* and *relation*.

A little time spent on each of these points for a few days will be sufficient preparation. A square yard subdivided into square feet should be drawn upon the board and remain there during the progress of this work.

NOTE.—Let the problem, $\frac{1}{9} + \frac{1}{3} =$ what? be written on the board. The pupil should use the pointer and clearly designate the *quantities* from the diagram as he proceeds with the analysis: One third of a square yard is three square feet; *one* square foot is one ninth of a square yard; *three* square feet are *three ninths* of a square yard; one ninth of a square yard and three ninths of a square yard are four ninths of a square yard. Therefore $\frac{1}{9} + \frac{1}{3} = \frac{4}{9}$.

NOTE.—Let the foregoing analysis guide in the solution of the following:—

$\frac{1}{9} + \frac{1}{3} = ?$	$\frac{2}{9} + \frac{1}{3} = ?$	$\frac{5}{9} + \frac{1}{3} = ?$
$\frac{1}{3} - \frac{1}{9} = ?$	$\frac{2}{3} - \frac{2}{9} = ?$	$\frac{5}{9} - \frac{1}{3} = ?$
$\frac{1}{3} \div \frac{1}{9} = ?$	$\frac{2}{3} \div \frac{2}{9} = ?$	$\frac{5}{9} \div \frac{1}{3} = ?$
$\frac{1}{9} \div \frac{1}{3} = ?$	$\frac{2}{9} \div \frac{2}{3} = ?$	$\frac{1}{3} \div \frac{5}{9} = ?$
$\frac{1}{9} \times 3 = ?$	$\frac{2}{3} \times \frac{1}{3} = ?$	$\frac{7}{9} \div \frac{2}{3} = ?$
$\frac{4}{9} \div \frac{1}{3} = ?$	$\frac{1}{2}$ of $\frac{4}{9} = ?$	$\frac{2}{3} \div \frac{7}{9} = ?$

CHAPTER X.

Thirteen—Fourteen.

LESSON 9—CLASS WORK.

Place on the table a gallon measure and a half gallon measure. Fill these with water. How many pints of water in the gallon measure? How many in the half gallon measure? How many in both? How much water in both? Now place on the table a pint measure. Fill it. How many pints of water now on the table? Twelve pints and one pint are how many pints? How many pints of water in the pint measure and the half-gallon measure? How many in the gallon measure? Five pints and eight pints are how many pints?

Now fill another pint measure, this time from the gallon measure. How many pints now in the gallon measure? How many pints of water in the other three measures? Seven pints of water and six pints of water are how many pints of water? Refill the gallon measure, using one of the pint measures. How many gallons and pints in thirteen pints? One pint is what part of a gallon? Five pints are what part of a gallon? Then how many gallons of water in thirteen pints? How many *eights* in thirteen? How many pints in a half gallon? How many half gallons and pints then on the table? How many *fours* of pints in thirteen pints?

Now fill two pint measures from the half-gallon measure. How many pints left in the half-gallon measure? How much water in the gallon measure and all the pint

measures? Eleven pints and two pints are how many pints? Empty one pint measure back into the half-gallon measure and set the empty measure away. How many pints in the gallon measure and the pint measures? How many in the half-gallon measure? Ten pints and three pints are how many pints?

DESK WORK.

$1\frac{1}{2}$ gal. + 1 pt. = pt.	13 pt. − 10 pt. = pt.
1 gal. + pt. = 13 pt.	8 pt. + 5 pt. = pt.
7 pt. + 6 pt. = gal.	13 pt. ÷ 1 gal. =
13 pt. − 11 pt. = pt.	13 pt. ÷ $\frac{1}{2}$ gal. =
9 pt. + pt. = 13 pt.	13 pt. ÷ 5 pt. =

LESSON 10--CLASS WORK.

Place on the table a gallon measure, two half-gallon measures, and four quart measures. Fill them with water. How many quarts of water in the gallon measure? How many in the half-gallon measures? How many in the quarts? How many in all? Three times four quarts are how many quarts? How many gallons? Using pint measures put enough more water on the table to make thirteen quarts. Twelve quarts and one quart are how many quarts? Group the gallon and pint measures into one group and the half-gallon and quart measures into another. How many quarts of water in the first group of measures? How many quarts of water in the second group of measures? Eight quarts of water and five quarts of water are how many quarts of water? How many gallons and quarts? How many gallons? Now put the four quart measures in the first group. How many quarts of water now in this group? How many in the second group? Nine quarts of water and four quarts of water are how many quarts of water? Now put the gallon measure and one half-gallon in one group and all the rest in another.

How many quarts of water in the first group of measures? How many in the second? Six quarts of water and seven quarts of water are how many quarts of water? How many *sixes* of quarts in thirteen quarts?

NOTE.—Let this be shown by grouping.

Now put the two pint measures and one half-gallon measure in one group and all the rest in another. How many quarts of water in this group? How many in this group? Three quarts of water and ten quarts of water are how many quarts of water?

DESK WORK.

12 qt. + 1 qt. = qt.
12 qt. + 1 qt. = gal. qt.
12 qt. + 1 qt. = gal.
10 qt. + 3 qt. = qt.
6 qt. + 6 qt. + 1 qt. = qt.
6 qt. + 7 qt. = qt.

8 qt. + 5 qt. = qt.
11 qt. + qt. = 3½ gal.
2½ gal. + qt. = 13 qt.
13 qt. ÷ 6 qt. =
13 qt. ÷ 4 qt. =
13 qt. ÷ 3 qt. =

LESSON 11—CLASS WORK.

NOTE.—Make such use of the measures in this lesson as may be deemed necessary. They should be before the class and used to some extent.

How many pecks make a bushel? In twelve pecks how many bushels? In thirteen pecks how many bushels? How many quarts make a peck? Put eight quarts in the half-bushel measure. Put five quarts in a peck measure. How much more in the half-bushel measure than in the peck measure? Five quarts are what part of a peck? The two measures have how many pecks in them? How many quarts? How many *fives* of quarts in this measure? How many *fives* of quarts in this measure? How many *fives* of quarts in both? How many *fives* of quarts in thirteen quarts? Take one quart out of the large measure and put it in the small one. How many now in the large measure?

In the small one? How many more in the large one? How many *sixes* of quarts in the small measure? In the large one? In both? How many *sixes* of quarts in thirteen quarts? How many quarts does the large measure contain? What part of *that* does the small one contain? How many *sevens* of quarts in the large measure? How many in the small one? How many in both? How many *sevens* of quarts in thirteen quarts?

NOTE.—So any other *parts* of thirteen may be compared with each other and with thirteen.

DESK WORK.

13 qt. = pk. qt.	13 qt. ÷ 5's of qt. =
13 qt. = pk.	13 qt. ÷ 6's of qt. =
13 pk. = bu. pk.	13 qt. ÷ 7's of qt. =
13 pk. = bu.	13 qt. ÷ 10's of qt. =
13 qt. ÷ 8's of qt. =	13 qt. ÷ 3's of qt. =

LESSON 12—CLASS WORK.

NOTE.—Furnish each child with fourteen one-inch sticks; call them one-inch measures.

You may hold up a one-inch measure. Now, count to find out how many one-inch measures I have given you. Make two lines with these measures just six inches long. Test the lines with your foot measures. How long is each? How many inch measures have you not used? How many have you used? Twelve inch-measures and two inch-measures are how many inch-measures? Now put one of the two you had left in the front line and the other in the back line. How many now in the front line? How long is the front line? How many in the back line? How long is the back line? Two times seven inches are how many inches? In fourteen inches how many *sevens* of inches? Seven inches and seven inches are how many inches?

Now take one inch-measure from the end of the back line and put it at the end of the front line. How long now is the back line? How long is the front line? Six inches and eight inches are how many inches? Take another from the back line and put it in the front line. How long now is the back line? The front line? Five inches and nine are how many inches? Put another from the back line into the front. How many now in the back line? How many in the front line? How long is each line? Four inches and ten inches are how many inches? Eleven inches and three inches are how many inches? Twelve inches and two inches? Thirteen inches and one inch?

DESK WORK.

7 in. + 7 in. = in.	9 in. + in. = 14 in.
14 in. ÷ 7 in. =	10 in. + in. = 14 in.
7 in. × 2 = in.	14 in. = ft. in.
8 in. + 6 in. = in.	14 in. − 3 in. = in.
12 in. + 2 in. = in.	14 in. − 7 in. = in.

LESSON 13—CLASS WORK.

NOTE.—Let a number of pupils stand before the class and hold the measures up in a line end to end.

Using foot measures, make a line fourteen feet long. How many foot measures have we used? How long is the line? Now break the line in the middle. How long is this part? How long is this part? How many *sevens* of feet in fourteen feet? Hold up higher one *seven* of feet. Hold higher the other *seven* of feet. Two times seven feet are how many feet? This part is how much of fourteen feet? This part is how much of fourteen feet? What is one half of fourteen feet? What are two halves of fourteen feet? Now make one line again of all the measures. How long is the line? Commencing at this end let us

make as many two-foot lines as we can. How many are there? Count them. In fourteen feet how many *twos* of feet? Hold up higher one *two* of feet. Two *twos* of feet. Three *twos* of feet. Three *twos* of feet are how many feet? Four *twos* of feet. Four *twos* of feet are how many feet? Five *twos* of feet. Five *twos* of feet are how many feet? Six *twos* of feet. Six *twos* of feet are how many feet? Seven *twos* of feet. Seven *twos* of feet are how many feet?

Five times two feet are how many feet?

Six times two feet are how many feet?

Seven times two feet are how many feet?

Again, make one straight line of all the measures. Break the line here; if this part is six feet long, how long is that part? Six feet and eight feet are how many feet? Break the line here; if this part is four feet long, how long is this part? Break it here; if this part is five feet long, how long is that part? Break here; if this line is three feet long, how long is that part? Three feet from fourteen feet are how many feet?

Desk Work.

7 ft. + 7 ft. = ft.
14 ft. ÷ 7 ft. =
7 ft. × 2 = ft.
½ of 14 ft. = ft.
⅔ of 14 ft. = ft.
6 ft. + 8 ft. = ft.
6 ft. + 7 ft. = ft.
14 ft. − 5 ft. = ft.
11 ft. + 3 ft. = ft.
11 ft. + 2 ft. = ft.
14 ft. − 9 ft. = ft.
5 ft. + ft. = 14 ft.
6 ft. + ft. = 14 ft.

8 ft. + ft. = 14 ft.
14 ft. − 4 ft. = ft.
4 ft. + ft. = 14 ft.
14 ft. ÷ 2 ft. =
2 ft. × 7 = ft.
12 ft. + 2 ft. = ft.
½ of 12 ft. = ft.
½ of 10 ft. = ft.
½ of 5 ft. = ft. in.
½ of 8 ft. = ft.
1 ft. + 2 in. = in.
14 in. − 8 in. = ft.
⅔ ft. + ½ ft. = in.

LESSON 14—CLASS WORK.

NOTE.—Let a number of pupils stand before the class to hold up the measures.

Using yard measures, make a line four yards long. How many yard measures? How many yards long? How many feet long? I will add another measure two feet long; how long is the line now? Twelve feet and two feet are how many feet? One foot is what part of a yard? These two feet are what part of a yard? How many yards long is the line? In fourteen feet how many yards? How many feet in one yard? Then how many *threes* of feet in fourteen feet? In fourteen feet how many yards? In thirteen feet (teacher substituting a foot-measure for the two-foot measure) how many yards and feet? How many yards? In twelve feet (teacher removing the one-foot measure) how many yards? In twelve feet how many *threes* of feet? In eleven feet (teacher substituting a two-foot measure for a yard measure) how many yards? How many *threes* of feet? In nine feet (removing the two-foot measure) how many yards? How many *threes* of feet? In ten feet (adding a foot-measure) there are how many yards? How many *threes* of feet? In eight feet? In six feet? In seven feet? In five feet? In three? In four? In two? In one?

DESK WORK.

3 yd. + 1 ft. = ft.
4 yd. + 1 ft. = ft.
4 yd. + 2 ft. = ft.
4 yd. + 2 ft. = yd.
$3\frac{2}{3}$ yd. = ft.
12 ft. ÷ 1-yd. =
12 ft. ÷ 3-ft. =
14 ft. ÷ 3-ft. =
14 ft. ÷ 1-yd. =
8 ft. = 1 yd. ft.
13 ft. ÷ 1-yd. =
6 ft. ÷ 3's of ft. =
7 ft. ÷ 1-yd. =
14 ft. ÷ 4's of ft. =

LESSON 15—CLASS WORK.

Place on the table a peck measure and four quart measures. Fill them with corn. How many pecks of corn on the table? How many quarts of corn on the table? How many *fours* of quarts are there? How many quarts must be added to make fourteen quarts? Twelve quarts and two quarts are how many quarts? How many pecks? How many *eights* of quarts? How many *fours* of quarts in fourteen quarts? Then how many *fours* of pecks in fourteen pecks? Then how many bushels in fourteen pecks?

DESK WORK

14 qt. ÷ 4-qt. =
14 pk. ÷ 1-bu. =
14 qt. ÷ 1-pk. =
14 qt. ÷ 8-qt. =
3 bu. + pk. = pk.
3 bu. + 2 pk. = bu.

1 pk. + 6 qt. = qt.
$\frac{3}{4}$ pk. + 8 qt. = pk.
12 pk. + $\frac{1}{2}$ bu. = pk.
13 pk. = bu. pk.
13 pk. = bu.
14 pk. = $\frac{1}{2}$-bu.

LESSON 16—CLASS WORK.

How many days in a week? How many *sevens* of days in fourteen days? How many weeks in fourteen days? Two times seven days are how many days? Seven days and seven days are how many days? Name the days of the week. How many Sundays in a week? How many Sundays in two weeks? How many Mondays in two weeks? How many Tuesdays? Wednesdays? Thursdays? Fridays? Saturdays? How many *twos* of days in fourteen days? Seven times two days are how many days? Who knows what *one* thing fourteen days make? How many weeks in a fortnight? One week is what part of a fortnight? How many work days in a week? In two weeks? If there are twelve work days in a fortnight, how

many rest days are there? Twelve days and two days are what? How many school days in a week? In two weeks? How many days that are not school days in a fortnight? Ten days and four days are how many days? If there are seven days in a week, one day is what part of a week? Two days? Three days? Four days? Five days? Six days? Seven days? Eight days are how many weeks? Nine days? Ten days? Eleven days? Twelve days? Thirteen days? Fourteen days?

DESK WORK.

7 da. + 7 da. = da.	8 da. = wk.
7 da. + 7 da. = wk.	10 da. = wk.
7 da. + 7 da. = ftnt.	9 da. = wk.
2 da. × 7 = wk.	12 da. = wk.
13 da. − 6 da. = wk.	11 da. = wk.
8 da. + 6 da. = wk.	13 da. = wk.
5 da. + da. = 14 da.	$\frac{1}{7}$ wk. + 13 da. = wk.

LESSON 17—CLASS WORK.

THE DIME AS A UNIT OF REFERENCE.

NOTE.—Let the problem, $\frac{1}{2}+\frac{2}{5}=$ what? be written on the board A dime, a half dime and several one-cent pieces are needed for illustrative purposes. Let them be placed on a flat surface in full view of the class and be used by the one solving the problem as he gives this analysis: One half of a dime is five cents (arranging five cent-pieces in a row to themselves); one cent (holding up the one cent) is one tenth of a dime; five cents, (holding up a half dime or nickel) are five tenths of a dime; two fifths of a dime are four cents; one cent is one tenth of a dime, and four cents are four tenths of a dime; five tenths of a dime and four tenths of a dime are nine tenths of a dime. Therefore, etc.

NOTE.—Adapt and apply the foregoing to these problems:—

$\frac{1}{2}+\frac{1}{5}=?$	$\frac{1}{2}\div\frac{2}{5}=?$	$\frac{1}{2}\div\frac{3}{5}=?$
$\frac{1}{2}-\frac{1}{5}=?$	$\frac{3}{5}+\frac{1}{2}=?$	$\frac{1}{2}+\frac{3}{10}=?$
$\frac{1}{2}\div\frac{1}{5}=?$	$\frac{3}{5}-\frac{1}{2}=?$	$\frac{3}{5}+\frac{7}{10}=?$
$\frac{1}{2}-\frac{2}{5}=?$	$\frac{3}{5}\div\frac{1}{2}=?$	$\frac{7}{10}\div\frac{3}{5}=?$

CHAPTER XI.

Twelfths—Fifteen—Fractions.

LESSON 18—CLASS WORK.

NOTE.—Let each pupil have a foot measure.

How many inches in a foot? Show me one inch. One inch is what part of a foot? Two inches? What other name for this part? Three inches? What other name for that part? Four inches? What other name? Five inches? Six inches? What other name? Seven inches? Eight inches? What other name? Nine inches? What other name? Ten inches? What other name? Eleven inches? Twelve inches? What is one half-foot in inches? How many inches in one third of a foot? In two thirds of a foot? In one fourth of a foot? In two fourths of a foot? In a half foot? In three fourths of a foot? In one sixth of a foot? In five sixths of a foot? In one twelfth of a foot? In five twelfths? In six twelfths? In eleven twelfths?

DESK WORK.

1 ft. = in.
$\frac{1}{2}$ of 12 in. = in.
12 in. ÷ 6 in. =
$\frac{1}{3}$ of 12 in. = in.
12 in. ÷ 4 in. =
$\frac{1}{4}$ of 12 in. = in.
12 in. ÷ 3 in. =
$\frac{1}{6}$ of 12 in. = in.
12 in. ÷ 2 in. =

5 in. × 2 = in.
4 in. × 3 = in.
3 in. × 4 = in.
$\frac{2}{3}$ of 12 in. = in.
$\frac{3}{4}$ of 12 in. = in.
$\frac{5}{6}$ of 12 in. = in.
$\frac{1}{2} = \frac{\ }{4} = \frac{\ }{6}$
$\frac{1}{6} = \frac{\ }{3}$
$\frac{2}{6} = \frac{\ }{3}$

Lesson 19—Class Work.

Place on the table a peck measure, a half-peck measure, and three quart measures. Now fill them with corn. Put in one group the peck measure and two quart measures of corn; in another group the half peck and the remaining quart of corn. How many quarts of corn in the first group? In the second group? Who can tell how many quarts of corn in both groups? How many five-quarts in fifteen quarts of corn. Five quarts of corn and five quarts of corn and five quarts of corn are how many quarts of corn? Two times five quarts of corn are how many quarts of corn? Three times five quarts of corn are how many quarts of corn? Place the peck measure and the half peck in one group, and the quart measures in another. How many quarts of corn in the first group? How many in the second? How many *threes* of quarts in the second? How many *threes* of quarts in the first? Four *threes* of quarts and one *three* of quarts are how many *threes* of quarts? How many quarts? In fifteen quarts there are how many *threes* of quarts? Four times three quarts are how many quarts? Five times three quarts are how many quarts? Count fifteen quarts by *threes*.

Fifteen quarts of corn are how many more than twelve quarts? How many *sixes* of quarts in twelve quarts? The remaining three quarts is what part of a six of quarts? Then in fifteen quarts how many sixes of quarts?

Place the peck measure by itself and all the other measures in one group. How many quarts in the peck measure? How many in the group of measures? Eight quarts and seven quarts are how many quarts.

Note.—Thus develop other *parts* of fifteen.

Desk Work.

15 qt. = pk. qt.	9 qt. + qt. = 1 pk. + 7 qt.
15 qt. = pk.	11 qt. + 4 qt. = pk.
8 qt. + qt. = 15 qt.	13 qt. + 2 qt. = qt.

15 qt. ÷ 6 qt. =
15 qt. ÷ 5 qt. =
15 qt. ÷ 3 qt. =
12 qt. + 3 qt. = qt.
10 qt. + qt. = 15 qt.

5 qt. × 3 = qt.
3 qt. × 5 = qt.
15 qt. ÷ 1 qt. =
15 pk. = bu. pk.
15 pk. = bu.

Lesson 20—Class Work.

Note.—Let each pupil have a yard measure.

Show me on the yard measure one foot. How many inches? Show me fifteen inches. Fifteen inches is how much more than twelve inches? One inch is what part of a foot? Three inches? In fifteen inches there are how many feet? Twelve inches and three inches are how many inches? How many feet? Show me, now, five inches; add to this five more inches; five inches and five inches are how many inches? Add five more inches; how many

inches are you showing now? How many *fives* of inches in fifteen inches? Three times five inches are how many inches? Divide fifteen inches into two as nearly equal numbers of inches as you can. What is the first number? What is the second? Seven inches and eight inches are how many feet? Count fifteen inches by *threes* of inches. Find how many *threes* of inches in fifteen inches; how many? How many *sixes* of inches in twelve inches? How many more inches would make fifteen inches? Three inches are what part of a *six* of inches? In fifteen inches there are how many sixes of inches? Show me ten inches; how many more inches would make fifteen inches? Five inches are what part of ten inches? In fifteen inches how many *tens* of inches? In twelve inches how many *fours* of inches? How many more inches would make fifteen inches? Three inches are what part of four inches? In fifteen inches how many *fours* of inches? Show me fourteen inches; how many *twos* of inches in fourteen inches? How many more inches would make fifteen inches? What part of two inches is one inch? In fifteen inches there are how many *twos* of inches? What is one half of fifteen inches? One third of fifteen inches? Two thirds of fifteen inches? What is one fifth of fifteen inches? Two fifths? Three fifths? Four fifths? Five fifths? How many *sevens* of inches in fourteen inches? How many more inches would make fifteen inches? One inch is what part of seven inches? How many sevens of inches in fifteen inches?

DESK WORK.

15 in. = ft. in.
15 in. = ft.
15 in. ÷ 5 in. =
$\frac{1}{3}$ of 15 in. = in.
$\frac{2}{3}$ of 15 in. = in.
5 in. × 2 = in.

$\frac{3}{5}$ of 15 in. = in.
$\frac{4}{5}$ of 15 in. = ft.
3 in. × 3 = in.
3 in. × 4 = ft.
3 in. × 5 = in.
14 in. ÷ 2 in. =

5 in. × 3 = ft.	7 in. + 8 in. = in.
15 in. ÷ 3 in. =	9 in. + 6 in. = ft.
15 in. ÷ 6 in. =	11 in. + in. = 15 in.
15 in. ÷ 10 in. =	15 in. − 5 in. = in.
15 in. ÷ 7 in. =	15 in. − 2 in. = ft.
$\frac{1}{5}$ of 15 in. = in.	15 in. − 6 in. = ft.
$\frac{2}{5}$ of 15 in. = ft.	15 in. − 7 in. = ft.

LESSON 21—CLASS WORK.

NOTE.—Let a number of pupils stand before the class and using foot measures make a line by placing them end to end horizontally, just fifteen feet long, holding the measures rather low

How long is this line of measures? Now, beginning at this end, raise this much (three feet) of the line higher. Class, how long is the part of the line just raised above the other part? In this way you may raise the whole line —three feet at a time—and *as you do so* the class may add by *threes* to the other end of the line. (Ans.: Three feet, six feet, nine feet, twelve feet, fifteen feet.) Lower the line and repeat. Lower the line again. Now *as the children raise the line—three at a time*—you may count the *threes* of feet. (Ans.: One *three*, two *threes*, three *threes*, four *threes*, five *threes*.) How many *threes* of feet in fifteen feet? Lower the line. This time we shall raise two feet at a time; first count the *twos*.

NOTE.—The *illustration* and *answer* should be given simultaneously.

(Ans.: One *two*, two *twos*, three *twos*, four *twos*, five *twos*, six *twos*, seven *twos*, and one-half *two*.) How many twos of feet in fifteen feet?

NOTE.—So take the *fours*, *fives*, and *sixes*.

DESK WORK.

12 ft. + ft. = 15 ft.	12 ft. ÷ 6's of ft. =
9 ft. + ft. = 15 ft.	15 ft. ÷ 6's of ft. =
15 ft. ÷ 3's of ft. =	9 ft. ÷ 6's of ft. =
12 ft. ÷ 3's of ft. =	$\frac{1}{3}$ of 15 ft. = ft.
15 ft. ÷ 2's of ft. =	$\frac{2}{3}$ of 15 ft. = ft.
10 ft. ÷ 5's of ft. =	$\frac{1}{5}$ of 15 ft. = ft.
15 ft. ÷ 5's of ft. =	$\frac{2}{5}$ of 15 ft. = ft.
12 ft. ÷ 4's of ft. =	$\frac{3}{5}$ of 15 ft. = ft.
15 ft. ÷ 4's of ft. =	$\frac{4}{5}$ of 15 ft. = ft.

LESSON 22—CLASS WORK.

NOTE.—Let five pupils stand before the class, each having a yard measure, holding it vertically.

Class, how many yard measures do you see? Place them in a horizontal straight line. How many yards long is this line? How many feet long is the line? In fifteen feet how many yards? In fifteen feet how many *threes* of feet? In twelve feet (teacher removing one yard measure) how many yards? In thirteen feet (teacher adding a foot measure) how many yards? How many *threes* of feet? In fourteen feet (teacher adding another foot measure) how many yards? How many *threes* of feet? In eleven feet (teacher removing another yard measure) how many yards? How many *threes* of feet? In ten feet (removing a foot measure) how many yards? How many *threes* of feet? In nine feet (removing another foot measure) how many yards? How many *threes* of feet? In eight feet how many yards? In six feet how many yards? In seven feet how many yards? In three feet? In four feet? In five feet? In no feet? In one foot?

DESK WORK.

5 yd. = ft.	$3\frac{1}{3}$ yd. = yd. ft
14 ft. = yd. ft.	$\frac{2}{3}$ yd. = ft.
14 ft. = yd.	$1\frac{2}{3}$ yd. = ft.
13 ft. = yd. ft.	3 ft. × 2 = yd.
13 ft. = yd.	2 yd. × 2 = ft.
15 ft. ÷ 3's of ft. =	3 ft. × 4 = yd.
12 ft. ÷ 3's of ft. =	1 yd. × 5 = ft.
13 ft. ÷ 3's of ft. =	3 ft. × 5 = yd.
14 ft. ÷ 3's of ft. =	1 yd. × 4 = ft.
$2\frac{2}{3}$ yd. = yd. ft.	15 ft. ÷ 10's of ft. =

LESSON 23—CLASS WORK.

Place on the table a gallon, a half gallon, and three pint measures. Fill them with water. Let us call the water *milk* to-day. How many pints of milk in the half gallon measure? How many in the gallon measure? How many in the pint measures? How many in the gallon and half gallon together? How many in all? Put the gallon of milk in a place to itself; put the other measures in a group. How many pints of milk in the gallon measure? How many in the group of measures? Then eight pints of milk and seven pints of milk are how many pints of milk? One pint of milk is what part of a gallon of milk? Seven pints are what part of a gallon of milk? Then how many gallons of milk in all. How many *eights* of pints in the gallon measure? In the other measures? In fifteen pints? In the gallon measure how many *sevens* of pints? In the other measures? In fifteen pints of milk how many *sevens* of pints of milk? Now place the gallon measure and half gallon measure in one group and the others in another. How many pints of milk in the first group of measures? In the second? How many in both? Twelve pints of milk and three pints of milk are how many pints of milk?

How many *threes* of pints in the pint measures? How many *threes* of pints in the other measures? Then how many *threes* of pints in fifteen pints? One pint is what part of twelve pints? Three pints of milk are what part of twelve pints of milk? How many *twelves* of pints in fifteen pints?

Fill *from the gallon measure* two more pint measures. Have we the same number of pints of milk? Put the gallon measure by itself; how many pints of milk does it contain? Arrange the measures to show another six pints of milk. How many pints are left? One pint is what part of six pints of milk? Three pints? In fifteen pints of milk how many *sixes* of pints?

Place the half gallon of milk to itself; how many pints of milk does it contain? Empty two of the pint measures back into the gallon measures. How many *fours* of pints in the half gallon measure? How many *fours* of pints in the gallon measure? How many in both? How many pints remain? One pint is what part of four pints? Three pints are what part of four pints? Then how many *fours* of pints in fifteen pints?

Show a *two* of pints of milk; how much milk is that? (Ans.: That is a quart of milk.) How many quarts of milk in the half gallon measure? How many *twos* of pints? How many quarts of milk in the gallon measure? How many twos of pints? How many *twos* of pints in fifteen pints of milk? How many quarts of milk in fifteen pints of milk? What is one half of fifteen pints of milk in pints? In quarts?

DESK WORK.

15 pt. = gal. qt. pt.	$1\frac{1}{2}$ qt. × 5 = pt.
15 pt. = gal.	15 pt. ÷ 5 pt. =
15 pt. = qt. pt.	15 pt. ÷ 3 pt. =
8 pt. + 7 pt. = gal.	15 pt. ÷ 4 pt. =
$1\frac{1}{2}$ gal. + $1\frac{1}{2}$ qt. = pt.	15 pt. ÷ 6 pt. =

LESSON 24—CLASS WORK.

THE FOOT AS A UNIT OF REFERENCE.

NOTE —Let the problem, $\frac{1}{3} + \frac{1}{4} =$ what? be written on the board. The child holds in his hand, in vertical position, a foot measure, giving the following analysis: One third of a foot (showing this length with the measure) is four inches; one fourth of a foot (showing this with the measure) is three inches; one inch (showing one inch on the measure) is one twelfth of a foot; four inches (showing four inches) are four twelfths of a foot, and three inches (showing three inches) are three twelfths of a foot; four twelfths of a foot and three twelfths of a foot are seven twelfths of a foot (showing seven inches. Therefore, etc.

NOTE.—Adapt and apply the foregoing analysis to the following:

$\frac{1}{3} - \frac{1}{4} =$ what?	$\frac{3}{4} + \frac{1}{6} =$	$\frac{2}{3} + \frac{1}{2} =$
$\frac{1}{3} \div \frac{1}{4} =$	$\frac{3}{4} - \frac{1}{6} =$	$\frac{2}{3} - \frac{1}{2} =$
$\frac{1}{4} \div \frac{1}{3} =$	$\frac{3}{4} \div \frac{1}{6} =$	$\frac{2}{3} \div \frac{1}{2} =$
$\frac{2}{3} + \frac{1}{4} =$	$\frac{1}{2} + \frac{5}{6} =$	$\frac{1}{4} + 1\frac{1}{2} =$
$\frac{2}{3} - \frac{1}{4} =$	$\frac{1}{2} - \frac{1}{6} =$	$1\frac{5}{2} - \frac{1}{6} =$

CHAPTER XII.

Sixteen—Review Drills—Combinations—Column Addition.

LESSON 25—CLASS WORK.

Place on the table a peck and a half-peck measure. Fill both with corn. How many quarts of corn do both contain? This is how much less than fifteen quarts? Add to what is already on the table, three quart measures of corn.

Find how much corn in the half-peck measure and quart measures.

NOTE.—This should be done in two ways:—

1. Integer Method.—In the half-peck measure there are four quarts; in the other measures there are three quarts; four quarts and three quarts are seven quarts.

2. Fractional Method.—One half peck is four quarts; one quart is one eighth of a peck, and four quarts are four eighths of a peck; three quarts are three eighths of a peck; four eighths of a peck and three eighths of a peck are seven eighths of a peck.

Seven eighths of a peck lacks how much of being a whole peck? Put one quart of corn more on the table. How many quarts did we have before? (Ans. Fifteen quarts.) How many now? Fifteen quarts and one quart are how many quarts?

How many *eights* of quarts in the peck measure? How many *eights* of quarts in all the other measures? How many pecks in all the measures? Eight quarts and eight quarts are how many quarts? In two pecks how many *eights* of quarts? In sixteen quarts how many eights of

quarts? Show with the measures how many one half pecks in a peck. Show with the measures a *four* of quarts. Show a measure having just a *four* of quarts in it. Then, the peck measure contains how many *fours* of quarts? In sixteen quarts how many *fours* of quarts? Two *fours* of quarts are how many pecks? Four *fours* of quarts are how many pecks? How many quarts?

Group the quart measures to show the *twos* of quarts in a half peck. Show another measure having just two *twos* of quarts in it. How many *twos* of quarts in the peck? In the two pecks? Then, in sixteen quarts how many *twos* of quarts are there?

Two times one peck are how many pecks? Two times one peck are how many quarts? How many half pecks in one peck? How many half pecks in two pecks? Four times one half peck are how many half pecks? Four times four quarts are how many quarts?

NOTE.—Review without the measures.

DESK WORK.

8 qt. + 7 qt. = qt.	2 pk. ÷ 8 qt. =
8 qt. + 8 qt. = qt.	2 pk. ÷ 4 qt. =
8 qt. × 2 = pk.	16 qt. ÷ 8 qt. =
8 qt. × 2 = qt.	16 qt. ÷ 4 qt. =
½ pk. ÷ 1 qt. =	4 qt. × 3 = qt.
1 pk. ÷ 4 qt. =	4 qt. × 4 = qt.
2 pk. ÷ ½ pk. =	16 qt. ÷ 2 qt. =

LESSON 26—CLASS WORK.

Place on the table a gallon, a half gallon, and four pint measures. Fill these measures with water. We shall call the water vinegar to-day. How many pints of vinegar in the gallon measure? In the half-gallon measure? In both? In the pint measures? In all?

Show a *four* of pints of vinegar (in pint measures). Show another four of pints of vinegar. How many *fours* in the gallon measure? A *four* of pints of vinegar is how much vinegar? How many half-gallons of vinegar on the table? How many gallons of vinegar? A half gallon of vinegar makes how many *fours* of pints of vinegar? Two half gallons of vinegar? Three half gallons? Four half gallons? In four half gallons of vinegar how many *fours* of pints of vinegar? In two gallons of vinegar how many *fours* of pints of vinegar? In sixteen pints of vinegar how many *fours* of pints of vinegar? Two times four pints are how many pints? Three times four pints? Four times four pints? How many gallons?

How many pints of vinegar in the pint measure and the half gallon measure? How many gallons? How many *eights* of pints? How many eights of pints in the gallon measure? How many eights of pints in all? How many *eights* of pints in sixteen pints? Two times eight pints are how many pints? Eight pints and eight pints are how many pints?

The *pint* measures contain how much vinegar? Group *them* to show how many quarts they contain. (Statement: The pint measures contain two quarts of vinegar.) How many *twos* of pints in this half gallon of vinegar? In the other half gallon of vinegar? In the gallon of vinegar? In the other gallon of vinegar? In the two gallons of vinegar how many *twos* of pints of vinegar? Then, in sixteen pints of vinegar, how many *twos* of pints of vinegar? Two times two pints are how many pints? Four times two pints? Three times two pints? Five times two pints? Six times two pints? Seven times two pints? Eight times two pints?

DESK WORK.

1 gal. × 2 = pt.	4 pt. × 3 = pt.
12 pt. ÷ ½ gal. =	4 pt. × 4 = gal.
16 pt. ÷ ½ gal. =	4 pt. × 4 = pt.
16 pt. ÷ 4 pt. =	16 pt. ÷ 2 pt. =
16 pt. ÷ 8 pt. =	2 pt. × 4 = gal.
8 pt. × 2 = gal.	2 pt. × 5 = pt.
8 pt. × 2 = pt.	2 pt. × 7 = pt.
4 pt. × 3 = gal.	2 pt. × 8 = gal.

LESSON 27—CLASS WORK.

NOTE.—Use the *same material* as in the last lesson.

How many pints of vinegar in the gallon measure? How many in the other measures? How many in all? Place one pint of vinegar with the gallon of vinegar. How many now in this group? (Answer: One pint from eight pints of vinegar leaves seven pints of vinegar.) How many in this group? (Ans. One pint and eight pints of vinegar are nine pints of vinegar.) Nine pints of vinegar and seven pints of vinegar are how many pints of vinegar? Place another pint of vinegar with the gallon measure. How many pints now in this group? How many in this one? Ten pints of vinegar and six pints of vinegar are

how many pints of vinegar? Change another pint. How many in this group? In this one? Eleven pints and five pints are how many pints? Change another pint measure. How many pints in this group? In this group? Twelve pints of vinegar and four pints of vinegar are how many pints of vinegar?

NOTE.—Thus develop all the parts of sixteen. Also, the parts may be compared with each other and with sixteen. For example:

This measure contains how much vinegar? (Ans. One half gallon.) These measures contain how much? (Ans. One and one half gallons.) The first quantity is what part of the second? Four pints of vinegar are what part of twelve pints of vinegar? The second quantity is how many times the first? Twelve pints are how many times four pints?

One half gallon is what part of two gallons? Four pints are what part of sixteen pints? One and one half gallons are what part of two gallons? Twelve pints are what part of sixteen pints?

DESK WORK.

8 pt. + 8 pt. = gal.
8 pt. + 8 pt. = pt.
9 pt. + 7 pt. = pt.
10 pt. + pt. = 16 pt.
2 gal. − 6 pt. = pt.
11 pt. + pt. = 2 gal.
2 gal. − 5 pt. = pt.
12 pt. + pt. = 2 gal.
16 pt. − 4 pt. = gal.
4 pt. = gal.
12 pt. − 8 pt. = gal.

12 pt. ÷ 4 pt. =
16 pt. ÷ 4 pt. =
4 pt. ÷ 12 pt. =
8 pt. ÷ 16 pt. =
10 pt. ÷ 6 pt. =
6 pt. ÷ 10 pt. =
16 pt. − 3 pt. = pt.
2 gal. − $\frac{1}{4}$ gal. = pt.
$\frac{3}{4}$ gal. + pt. = 16 pt.
$1\frac{1}{2}$ gal. + pt. = 2 gal.
2 gal. ÷ $\frac{1}{2}$-gal. =

LESSON 28—CLASS WORK.

NOTE.—For additional "Class Work" refer to lessons in Chapters IX, X., and XI, in which long measure is employed. Prepare the class for the following.—

DESK WORK.

12 in. ÷ 1 ft. =
16 in. ÷ 12 in. =
12 in. ÷ 6 in. =
16 in. ÷ 6 in. =
13 in. ÷ 6 in. =
14 in. ÷ 6 in. =
15 in. ÷ 6 in. =
15 in. ÷ 3 in. =
16 in. ÷ 3 in. =

12 in. ÷ 4 in. =
16 in. ÷ 4 in. =
16 in. ÷ 8 in. =
16 in. ÷ 2 in. =
4 in. × 3 = ft.
4 in. × 4 = ft.
2 in. × 8 = in.
$\frac{1}{2}$ of 16 in. = in.
$\frac{1}{2}$ of 14 in. = in.

DESK WORK.

12 ft. ÷ 1 yd. =
15 ft. ÷ 1 yd. =
16 ft. = yd. ft.
16 ft. = yd.
13 ft. = yd. ft.
14 ft. = yd. ft.
9 ft. ÷ 3 ft. =
4 ft. × 3 = ft.
4 ft. × 3 = yd.
4 ft. × 4 = ft.
4 ft. × 4 = yd.
8 ft. × 2 = ft.

4 yd. + 4 ft. = ft.
5 yd. + ft. = 16 ft.
10 ft. + ft. = 16 ft.
12 ft. ÷ 4 ft. =
16 ft. ÷ 4 ft. =
16 ft. ÷ 8 ft. =
$\frac{1}{2}$ of 16 ft. =
$\frac{1}{4}$ of 16 ft. =
$\frac{3}{4}$ of 16 ft. =
$\frac{1}{8}$ of 16 ft. =
$\frac{5}{8}$ of 16 ft. =
$\frac{7}{8}$ of 16 ft. =

LESSON 29—CLASS WORK.

Children, if we wish to know "how much" corn in a quantity of corn, what units of measure do we use? What is the *principal* unit? If we wish to measure wheat or apples or potatoes, etc., what do we use?

If we wish to find out "how much" milk or molasses or vinegar or oil, etc., what units are used? What is the principal unit?

If we wish to find out "how much" length anything has, what units are used? Which are the principal ones?

If we wish to know "how much" sugar or rice or starch or salt, etc., what do you think would be the principal unit to use?

We have on the table to-day a pair of scales. This is an instrument used to tell how many pounds in any quantity.

This is a pound weight. If a quantity of sugar is just as heavy as the weight is, or just balances the weight, how much sugar would there be? This is a half-pound weight; how much salt will it take to balance it? I see the figure "1" on the pound weight; what do you think that means? Now, on the half-pound weight, I see the figure "8"; what do you think that means? How many ounces then in a half pound? If eight ounces make a half pound, how many ounces in a pound? If eight ounces make a half pound, what would four ounces be? Who can pick out a one-fourth-pound weight? What figure do you find on it? What does this four mean? How many of these would equal a half-pound weight? How many a pound weight? Is the ounce weight larger or smaller than this? How do you know? Do you think now you can pick out an ounce weight?

Place the one-pound weight at one end of the scales and enough ounces at the other end to balance it. How many ounces are required? How many eight-ounce weights equal a pound weight? Prove it. How many four-ounce weights equal a pound weight? Prove it. Pick out a two-ounce weight. Find how many of these equal a pound. How many eight ounces in a pound? How many four ounces in a pound? How many two ounces in a

pound? Eight ounces and eight ounces are how many ounces? Two times eight ounces are how many ounces? Eight times two ounces are how many ounces? Four times four ounces are how many ounces? Three times four ounces? Two times four ounces? One half of sixteen ounces are how many ounces? Two halves of sixteen ounces are how many? One fourth of sixteen ounces are how many ounces? Two fourths? Three fourths? Four fourths? One eighth of sixteen ounces are how many ounces? Three eighths? Four eighths? Five eighths? Seven eighths? Eight eighths?

DESK WORK.

1 lb. = oz.	16 oz. ÷ 8 oz. =
½ lb. = oz.	16 oz. ÷ 4 oz. =
¼ lb. = oz.	16 oz. ÷ 2 oz. =
⅛ lb. = oz.	12 oz. ÷ 4 oz. =
¾ lb. = oz.	4 oz. × 2 = oz.
⅜ lb. = oz.	4 oz. × 4 = oz.
⅝ lb. = oz.	4 oz. × 3 = oz.
⅞ lb. = oz.	2 oz. × 6 = lb.
8 oz. + 8 oz. = lb.	2 oz. × 8 = oz.
16 oz. − 8 oz. = lb.	12 oz. + ¼ lb. = lb.
8 oz. × 2 = oz.	2 oz. × 5 = oz.

4 oz. + 4 oz. + 4 oz. + 4 oz. = lb.

LESSON 30—CLASS WORK.

NOTE.—Let the object of this lesson be *facility* in weighing and *review* of last lesson.

Weigh a pound of salt. Show the class how much a pound of salt is. Put it in a paper bag. Who else would like to weigh a pound of salt? Show it to the class. Now, take the weight off the scales and see how closely you "can come" to a pound. Test it with the weight. Have you too much or too little? Make it right.

Weigh a half pound of salt. What does this lack of being a pound? How many half pounds in a pound? Weigh one fourth of a pound of salt. Show the class. Which is more, one fourth of a pound or one half of a pound? How many one-fourth pounds in one-half pound? How many in a whole pound?

Weigh one eighth of a pound. Show it to the class. Which is greater, one eighth or fourth of a pound? How many one eighths in one fourth? In one half? In one whole pound? How many ounces in a pound? In a half pound? In one fourth of a pound? In one eighth of a pound?

Weigh three fourths of a pound. What does this lack of being a pound? How many one fourths of a pound in three fourths of a pound? One fourth of a pound is what part of three fourths of a pound?

Weigh three eighths of a pound. Weigh five eighths of of a pound. Is this more or less than a half pound? Weigh seven eighths of a pound. What weight would you have to put on now to make a pound?

DESK WORK.

NOTE.—Select the more difficult problems from the last desk lesson for this lesson.

Drills to be placed on blackboard for frequent recitation:

1.	1 gal.	=	pt.	
	2 gal.			
	$1\frac{1}{2}$ gal.			
	$1\frac{3}{4}$ gal.			
	$1\frac{1}{2}$ gal.			
	$1\frac{3}{8}$ gal.			
	$1\frac{5}{8}$ gal.			
	$1\frac{7}{8}$ gal.			
	$\frac{1}{2}$ gal.			
	$\frac{3}{4}$ gal.			
	$\frac{3}{8}$ gal.			
	$1\frac{1}{4}$ gal.			

2.	10 pt.	=	qt.	
	14 pt.			
	12 pt.			
	1 pt.			
	13 pt.			
	15 pt.			
	16 pt.			
	7 pt.			
	9 pt.			
	8 pt.			
	12 pt.			
	11 pt.			
	15 pt.			

3. 4 bu.
2 bu.
$2\frac{1}{2}$ bu.
1 bu.
$1\frac{1}{4}$ bu.
$1\frac{1}{2}$ bu.
3 bu.
$1\frac{3}{4}$ bu.
$2\frac{3}{4}$ bu.
$3\frac{1}{2}$ bu.
$3\frac{1}{4}$ bu.
= pk.

4. 8 qt.
16 qt.
12 qt.
9 qt.
10 qt.
13 qt.
15 qt.
14 qt.
11 qt.
7 qt.
6 qt.
= pk.

5. 1 yd.
3 yd.
2 yd.
4 yd.
$2\frac{1}{2}$ yd.
$1\frac{1}{2}$ yd.
$3\frac{1}{2}$ yd.
$4\frac{1}{2}$ yd.
5 yd.
$5\frac{1}{3}$ yd.
$4\frac{2}{3}$ yd.
$3\frac{2}{3}$ yd.
= ft.

6. 12 ft.
9 ft.
6 ft.
3 ft.
15 ft.
14 ft.
13 ft.
10 ft.
11 ft.
= yd.

NOTE.—The teacher should change these forms from time to time and construct others. Also, give drills on all "combinations" to 16.

COLUMN ADDITION.

7.	8.	9.	10.	11.	12.
3	4	6	5	4	2
2	1	1	2	2	2
4	5	2	1	2	2
1	2	4	2	2	2
2	1	0	3	2	1
3	3	1	2	2	0
1	0	1	0	2	4

Lesson 31—Class Work.

The Fortnight as a Unit of Reference.

Note.—Write the problem, $\frac{1}{14} + \frac{3}{7} =$ what? on the board and require this analysis: Three sevenths of a fortnight are *six* days; one day is one fourteenth of a fortnight, and six days are six fourteenths of a fortnight, one fourteenth of a fortnight and six fourteenths of a fortnight are seven fourteenths of a fortnight or one half of a fortnight. Therefore, etc.

Note.—Have pupils solve and similarly explain the following —

$\frac{1}{2} + \frac{1}{14} =$ $\quad$ $\frac{1}{2} \div \frac{5}{14} =$ $\quad$ $\frac{3}{14} + \frac{1}{2} =$

$\frac{1}{2} + \frac{1}{7} =$ $\quad$ $\frac{1}{2} \div \frac{5}{7} =$ $\quad$ $\frac{9}{14} + \frac{1}{2} =$

$\frac{1}{2} - \frac{1}{7} =$ $\quad$ $\frac{5}{7} + \frac{3}{14} =$ $\quad$ $\frac{9}{14} - \frac{1}{2} =$

$\frac{1}{2} - \frac{3}{7} =$ $\quad$ $\frac{5}{7} \div \frac{3}{14} =$ $\quad$ $\frac{9}{14} \div \frac{1}{2} =$

$\frac{1}{2} \div \frac{3}{7} =$ $\quad$ $\frac{3}{14} \div \frac{5}{7} =$ $\quad$ $\frac{1}{2} - \frac{1}{14} =$

$\frac{1}{2} \div \frac{3}{14} =$ $\quad$ $\frac{3}{14} \div \frac{1}{2} =$ $\quad$ $\frac{11}{14} \div \frac{5}{7} =$

CHAPTER XIII.

Practical Problems—Seventeen.

LESSON 32—CLASS WORK.

1. How many pounds of nails can be bought for 8 cents if they cost 4 cents a pound? For 16 cents? For 12 cents? For 14 cents? For 10 cents? For 9 cents? For 13 cents? For 15 cents?

2. A pound of wire nails is worth 5 cents; what are two pounds worth? Three pounds? One and one half pounds? Two and one half pounds?

3. A pound of stick candy is worth 16 cents; how much stick candy will 8 cents buy? 4 cents? 12 cents? 1 cent? 3 cents? 2 cents? 5 cents? 10 cents? 14 cents?

4. A boy had 8 cents in one pocket and 8 cents in another, how much did he have in both? How many nickels and cents would this be equal to? How many dimes, nickels and cents? How many dimes? If he took 2 cents out of one pocket and put it in the other, how much would he have in each pocket? In all? If he took 3 cents out, etc.?

5. If apples cost 15 cents a dozen, what would be the cost of 4 apples? 8 apples? 6 apples? 10 apples? 2 apples?

6. If oranges are worth 16 cents a dozen, what will be the cost of three oranges? 6 oranges? 9 oranges?

7. A lady bought a pound of sugar for 8 cents; she spilt 4 ounces on the way home, how many ounces did she

have left? What part of a pound did she have left? How much more did she have left than she lost? What she had left was how many times as much as she lost? What she lost was what part of what she had left? What was the part she lost worth? What was the other part worth?

8. If 4 ounces of sugar cost 2 cents, what ought one ounce to cost? What then will 3 cents buy? 5 cents? 6 cents? $2\frac{1}{2}$ cents? 7 cents? $5\frac{1}{2}$ cents?

9. $\frac{1}{2}$ of 14 ounces and 9 ounces are what?

10. 14 ounces and how many ounces make a pound?

Lesson 33—Class Work.

1. If $\frac{1}{2}$ gallon of gasoline costs 12 cents what will 1 pint cost? 1 quart? $\frac{3}{8}$ gallon? $2\frac{1}{2}$ quarts? $\frac{3}{4}$ gallon?

2. If 1 quart of gasoline costs 6 cents, how much gasoline can be bought for 9 cents? 3 cents? 15 cents? 12 cents?

3. If 1 pint of molasses costs 5 cents how much can be bought for 10 cents? For 15 cents?

4. At 5 cents per quart how much milk can be bought for 15 cents? For 10 cents?

5. If $\frac{1}{2}$ gallon of vinegar costs 10 cents what will 1 quart cost? $\frac{3}{4}$ gallon?

6. At 10 cents a quart what will 1 pint of molasses cost? 3 pints?

7. In a can was a gallon of milk; 3 pints leaked out. How many pints were left? What part of a gallon was left?

8. If $\frac{1}{2}$ gallon of coal oil costs 10 cents what will $\frac{1}{4}$ gallon cost? 3 quarts?

9. How much coal oil can be bought for 15 cents if 1 quart costs 5 cents?

10. If a lamp burns $\frac{1}{2}$ pint of coal oil in one night how much will it burn in a week?

LESSON 34—CLASS WORK.

1. Calico costs 6 cents per yard. How much will 2 yards cost? $\frac{1}{3}$ yard? $2\frac{1}{3}$ yards? $\frac{2}{3}$ yard? $1\frac{2}{3}$ yards? $\frac{1}{2}$ yard?

2. If lawn costs 6 cents per yard how many yards can be bought for 12 cents? 3 cents? 15 cents? 9 cents? 2 cents? 8 cents?

3. At 12 cents a yard what will $\frac{1}{2}$ yard of ribbon cost? 1 foot? $\frac{2}{3}$ yard? $1\frac{1}{3}$ yards? $\frac{1}{4}$ yard? $\frac{3}{4}$ yard? $1\frac{1}{4}$ yards? $\frac{1}{6}$ yard? $\frac{5}{6}$ yard?

4. At 16 cents a yard how much ribbon can be bought for 8 cents? 4 cents? 12 cents? 2 cents? 6 cents? 10 cents? 14 cents?

5. If silk costs $2.00 per yard how much will $\frac{1}{2}$ yard cost? $1\frac{1}{2}$ yards? $2\frac{1}{2}$ yards?

6. If 12 yards of dress goods cost $6 what will 6 yards cost? 2 yards? 10 yards? 1 yard? 4 yards? 8 yards? 16 yards?

7. A lady bought 12 yards of goods for a dress. This was $1\frac{2}{3}$ yards too much. How much did it take for the dress?

8. What will $\frac{2}{3}$ yard of ribbon cost at 15 cents per yard?

9. Nellie had 5 yards of ribbon; she cut off 2 yards. How many yards were in the long piece? How many feet? How many feet in the short piece? The short piece is how much of the long piece? How many feet in both pieces together?

10. In a piece of goods there were 12 yards; 8 yards were sold. How many yards were left? What part of the whole was sold? What part of the whole was left? The part left was what part of the piece sold?

LESSON 35—CLASS WORK.

1. If $\frac{1}{2}$ pound of sugar costs 3 cents what will 1 pound cost? $1\frac{1}{2}$ pounds? 2 pounds?

2. If $\frac{3}{4}$ pound of rice costs 6 cents what will 1 pound cost?

3. If rice costs 8 cents per pound what will 2 pounds cost? $\frac{1}{2}$ pound? $1\frac{1}{2}$ pounds? $\frac{1}{4}$ pound? $1\frac{1}{4}$ pounds? $\frac{3}{4}$ pound?

4. If $\frac{1}{2}$ pound of tea costs 14 cents what will $\frac{1}{4}$ pound cost?

5. If coffee costs 12 cents per pound how much will $\frac{1}{2}$ pound cost? $\frac{3}{4}$ pound? $1\frac{1}{4}$ pounds?

6. If butter costs 16 cents per pound how much will 6 ounces cost? $\frac{3}{4}$ pound? $\frac{7}{8}$ pound?

7. If rice costs 8 cents per pound how much can be bought for 16 cents? For 4 cents? For 12 cents? 2 cents? 6 cents? 10 cents?

8. If a pound of sugar costs 6 cents how much can be bought for 12 cents? 9 cents? 15 cents?

9. If it takes 1 ounce of coffee to make two cups, how many cups will 6 ounces make? $\frac{1}{4}$ pound? $\frac{1}{2}$ pound? 3 ounces? 2 ounces?

10. A little girl bought a pound of candy; she ate $\frac{3}{8}$ of it. How many ounces did she eat? How many ounces did she have left? What part of a pound did she have left?

Lesson 36—Class Work.

Place on the table a gallon, a half gallon, and two quart measures. Fill them with water. How many pints of water in all? I will add another pint of water to that already on the table; how many now? *Arrange* the measures to show how many gallons of water on the table. How many pints in a gallon? How many *eights* of pints in seventeen pints?

How many *half gallons* of water in this gallon of water? In this gallon of water? In this pint of water? Then how many *fours* of pints in this gallon? In this gallon? In this pint? In all? In seventeen pints there are how many fours of pints?

How many quarts of water in this gallon of water? How many *twos* of pints? How many twos of pints in this gallon? In this pint? In all? In seventeen pints of water there are how many *twos* of pints of water?

Now place in one group the gallon measure and the pint measure and in another all the others. How many pints of water in each group? Nine pints of water and eight pints of water are how many pints of water? Place the gallon and one quart measure in one group and the rest in another. How many pints of water in each? Ten pints and five pints of water are how many pints of water? Change the pint measure to the other group. How many pints now in each group? Eleven pints and six pints are how many pints? Put the two quarts in one group and all the rest in another. How many pints in each? Thirteen pints and four pints are how many pints? Change the pint measure to the other group. How many pints in each group? Twelve pints and five pints are how many pints?

DESK WORK.

16 pt. + 1 pt. = pt.

1½ gal. + qt. = 2 gal. + 1 pt.

16 pt. ÷ 8 pt. =

1 pt. ÷ 8 pt. =

17 pt. ÷ 8 pt. =

17 pt. ÷ 1 gal. =

1 gal. ÷ ½ gal. =

2 gal. ÷ ½ gal. =

1 pt. ÷ 4 pt. =

17 pt. ÷ ½ gal. =

17 pt. ÷ 4 pt. =

1 qt. ÷ 2 pt. =

1 gal. ÷ 2 pt. =

2 gal. ÷ 2 pt. =

1 pt. ÷ 2 pt. =

17 pt. ÷ 2 pt. =

16 pt. ÷ 1 qt. =

17 pt. ÷ 1 qt. =

9 pt. + 8 pt. = gal.

9 pt. + 8 pt. = pt.

10 pt. + 5 pt. = pt.

1⅜ gal. + 3 qt. = pt.

12 pt. + pt. = 17 pt.

4 pt. + 13 pt. = pt.

LESSON 37—CLASS WORK.

Place on the table a peck, a half peck, and five quart measures. Fill them with oats. How many quarts in the peck and half-peck measures? In the other measures? Twelve quarts of oats and five quarts of oats are how many quarts of oats? Place together the peck measure and one quart measure. How many quarts? How many quarts in the other measures? Nine quarts and eight quarts are how many quarts? Place together the peck and all the quart measures of oats; how many quarts of oats? How many in the other measure? Thirteen quarts of oats and four quarts of oats are how many quarts of oats?

Place two of the quart measures with the half-peck measure; how many quarts? How many in the other measures? Six quarts of oats and eleven quarts of oats are how many quarts of oats? Change another quart to the first group; how many quarts? How many in this? Seven quarts and ten quarts of oats are how many quarts of oats? Place three quart measures in one group and all the rest in another; how many in each? Three quarts and fourteen quarts are how many quarts? Put all but two quarts in one group. Two quarts and fifteen quarts are how many quarts?

Arrange the measures to show how many pecks in seventeen quarts; how many? How many *eights* of quarts?

Arrange the measures the best you can to show how many half pecks in seventeen quarts; how many? How many *fours* of quarts? How many pecks in a bushel? Then, how many bushels in seventeen pecks?

DESK WORK.

12 qt. + 5 qt. = qt.	16 qt. ÷ 8 qt. =
$1\frac{1}{2}$ pk. + 5 qt. = qt.	17 qt. ÷ 8 qt. =
2 pk. + qt. = 17 qt.	16 qt. ÷ 4 qt. =
$\frac{1}{2}$ pk. + qt. = $2\frac{1}{2}$ pk.	17 qt. ÷ 4 qt. =

1 pk. + 9 qt. = pk.
1 pk. + 9 qt. = qt.
13 qt. + ½ pk. = pk.
13 qt. + ½ pk. = qt.
1⅜ pk. + 6 qt. = qt.
1¼ pk. + qt. = 17 qt.
17 qt. − 4 qt. = qt.
15 qt. + qt. = 17 qt.

17 qt. ÷ ½ pk. =
1 bu. ÷ 1 pk. =
1 bu. ÷ 4 pk. =
8 pk. ÷ 1 bu. =
16 pk. ÷ 1 bu. =
16 pk. ÷ 4 pk. =
17 pk. ÷ 4 pk. =
17 pk. ÷ 1 bu. =

Lesson 38—Class Work.

Here is a yard measure; how many feet long is it? John may add to this another yard measure. How many yards long is the line of measures now? How many feet long? Ida may add another yard measure; how many yards long is the line now? How many feet long? How many *threes* of feet in the line? James may add another yard; how many yards long now? How many feet long? How many *threes* of feet in the line? Mary may add another yard measure; how many yards long now? How many feet long? How many *threes* of feet in the line now? Hugh may now add a foot measure. A foot is what part of a yard? How many yards long is the line now? How many *threes* of feet in the line? How many feet long? Julia may add another foot measure. Two feet are what part of a yard? How many feet long is the line of measures now? How many yards long? How many *threes* of feet in the line? In seventeen feet there are how many *threes* of feet? In sixteen feet (teacher removing one foot measure) there are how many yards? In fifteen feet (removing the other foot measure) there are how many yards? In fourteen feet (removing a yard measure and substituting two foot measures) there are how many yards? In thirteen feet (removing a foot measure) there are how many yards? In twelve feet (removing the other

foot measure) there are how many yards? In eleven feet there are how many yards? In ten feet? In nine feet? In eight feet? In seven feet? In six feet? In five feet? In four feet? In three feet? In two feet? In one foot?

DESK WORK.

6 ft. ÷ 3 ft. =	16 ft. ÷ 3's of ft. =
6 ft. ÷ 1 yd. =	17 ft. ÷ 3's of ft. =
6 ft. + 1 yd. = ft.	16 ft. ÷ 1 yd. =
9 ft. ÷ 1 yd. =	17 ft. ÷ 1 yd. =
9 ft. ÷ 3 ft. =	15 ft. ÷ 5's of ft. =
3 ft. × 3 = ft.	16 ft. ÷ 5's of ft. =
$\frac{2}{3}$ of 9 ft. = yd.	17 ft. ÷ 5's of ft. =
3 ft. × 4 = ft.	$\frac{3}{5}$ of 15 ft. = yd.
3 ft. × 4 = yd.	$5\frac{1}{3}$ yd. = ft.
$\frac{1}{2}$ of 12 ft. = ft.	$5\frac{2}{3}$ yd. = ft.
$\frac{3}{4}$ of 12 ft. = yd.	14 ft. = yd. ft.
5 ft. × 3 = yd.	17 ft. = yd. ft.

LESSON 39—CLASS WORK.

Weigh one pound and one ounce of salt; put it in a paper bag. How many ounces of salt in the bag? Place it on the table. Weigh just one pound of salt; how many ounces of salt does it contain? Place it beside the first bag. Weigh fifteen ounces of salt; put it in a bag and place it beside the last package. Weigh fourteen ounces. Weigh twelve ounces. How many packages of salt now on the table? Now, weigh a package of two ounces; take this in your left hand and the second one in your right hand; how many *twos* of ounces of salt in seventeen ounces of salt? (Ans. In sixteen ounces of salt there are eight *twos* of ounces of salt, and in seventeen ounces of salt there are eight and one-half *twos* of ounces of salt.)

Now, weigh a package of three ounces of salt. Take this in one hand and the third package in the other. How

many *threes* of ounces of salt in seventeen ounces of salt? (Ans. In fifteen ounces of salt there are five *threes* of ounces of salt, and in seventeen ounces of salt there are five and two-third *threes* of ounces of salt.) Now, weigh a four-ounce package. How many *fours* of ounces in seventeen ounces? (Ans. In sixteen ounces there are four *fours* of ounces, and in seventeen ounces there are

four and one-fourth *fours* of ounces.) How many *fives* of ounces in seventeen ounces? How many *sixes* of ounces in seventeen ounces? How many *sevens* of ounces? How many *eights* of ounces?

How many pounds and ounces in seventeen ounces? How many half pounds in seventeen ounces? How many one-fourth pounds? How many one-eighth pounds?

DESK WORK.

1 lb. ÷ 2 oz. =	17 oz. ÷ 7 oz. =
17 oz. ÷ 2 oz. =	12 oz. ÷ 6 oz. =
15 oz. ÷ 3 oz. =	16 oz. ÷ 6 oz. =

1 lb. ÷ 3 oz. =

16 oz. ÷ $\frac{1}{4}$ lb. =

17 oz. ÷ 4 oz. =

15 oz. ÷ 5 oz. =

1 lb. ÷ 5 oz. =

17 oz. ÷ 5 oz. =

17 oz. ÷ 3 oz. =

16 oz. ÷ 8 oz. =

17 oz. ÷ 8 oz. =

14 oz. ÷ 7 oz. =

17 oz. ÷ 6 oz. =

2 oz. × $\frac{1}{2}$ = oz.

2 oz. × $8\frac{1}{2}$ = oz.

5 oz. × 3 = oz.

5 oz. × $3\frac{2}{5}$ = oz.

$\frac{1}{2}$ of 16 oz. = lb.

$\frac{1}{4}$ of 1 lb. = oz.

$\frac{3}{4}$ of 1 lb. = oz.

$\frac{1}{8}$ of 1 lb. = oz.

$\frac{5}{2}$ of 1 lb = oz.

CHAPTER XIV.

Eighteen—Fractions—Review Drills.

LESSON 40—CLASS WORK.

Place on the table a peck measure, two half-peck measures, and one quart measure. Fill them with corn. How many quarts of corn in the peck measure? In the two half-peck measures? In both? How many quarts of corn on the table? I will add another quart of corn to that already on the table; how many are there now? Then, this is the number (18) I wish you to learn something about to-day. In sixteen quarts of corn there are how many pecks of corn? Arrange the measures to show this. How many quarts are left? Sixteen quarts of corn and two quarts of corn are how many quarts of corn? Place one of the quarts of corn upon the peck measure and the other upon the half-peck measures. How many quarts of corn in each quantity? How many *nines* of quarts of corn in eighteen quarts of corn? Nine quarts of corn and nine quarts of corn are how many quarts of corn? Two times nine quarts of corn are how many quarts of corn? What is one half of eighteen quarts of corn? How many pecks of corn in this quantity? In this quantity? One and one eighth pecks and one and one eighth pecks are how many quarts? Are how many pecks?

Now, place in one group the two quart measures of corn; this is a *two* of quarts of corn. How many twos of quarts in the peck measure? In the two half-peck meas-

ures? How many *twos* of quarts of corn on the table? How many *twos* of quarts of corn in eighteen quarts of corn? Four times two quarts of corn are how many quarts of corn? Eight times two quarts of corn? Nine times two quarts of corn?

Now, place upon the peck measure one of the half-peck measures of corn; how many quarts of corn in the two measures? Place the two quarts of corn upon the other half-peck measure; how many quarts do they contain? Then we will call it a *six* of quarts of corn. In the twelve quarts of corn there are how many *sixes* of quarts of corn? How many in all? Twice six quarts of corn are how many quarts of corn? Three times six quarts of corn are how many quarts of corn? What is one third of eighteen quarts of corn? Two thirds? In this *six* of quarts how many *threes* of quarts of corn? In this twelve of quarts how many *threes* of quarts? In eighteen quarts how many *threes* of quarts of corn? Six times three quarts of corn are how many quarts of corn?

This *six* of quarts is what part of the *twelve* of quarts? How many *twelves* of quarts in eighteen quarts?

Now, place all the measures except the peck measure in one group; how many *fours* of quarts of corn in it? How many *fours* of quarts in the peck measure? How many *fours* of quarts in eighteen quarts? Eight quarts of corn and ten quarts of corn are how many quarts of corn?

Get two empty quart measures and put them on the table; fill one from the peck measure and the other from one of the half-peck measures. Can you show me a *seven* of quarts of corn? Can you show me another *seven*? What part of a *seven* of quarts are the four remaining quarts of corn? In eighteen quarts of corn how many *sevens* of quarts of corn? Fill up the half peck measure again from one of the quart measures and put the empty measure away. Put the three quart measures in one group and all the rest in another; how many quarts of corn in this quan-

tity? In this quantity? In fifteen quarts of corn how many *fives* of quarts of corn? This quantity is what part of a *five* of quarts? Then, in eighteen quarts of corn there are how many *fives* of quarts of corn?

DESK WORK.

2 pk. + qt. = 18 qt.	9 qt. × 2 = pk.
18 qt. − 3 qt. = qt.	6 qt. × 3 = qt.
12 qt. + 6 qt. = qt.	15 qt. ÷ 5's of qt. =
18 qt. − 8 pk. = qt.	18 qt. ÷ 5's of qt. =
18 qt. ÷ 9's of qt. =	18 qt. = pk. qt.
½ of 18 qt. = pk.	14 qt. = pk. qt.
18 qt. ÷ 2's of qt. =	14 qt. ÷ 7's of qt. =
18 qt. ÷ 6's of qt. =	18 qt. ÷ 7's of qt. =
⅓ of 18 qt. = pk.	16 qt. ÷ 8's of qt. =
⅔ of 18 qt. = pk.	16 qt. ÷ 1 pk. =
18 qt. ÷ ½-pk. =	18 qt. ÷ 8's of qt. =
18 qt. ÷ 4's of qt. =	18 qt. ÷ 1's of pk. =

LESSON 41—CLASS WORK.

NOTE.—Let each pupil have a yard measure.

Show me on the yard measure one foot. Show me six inches. Show me three inches. Show me fifteen inches. Fifteen inches is how much more than one foot? Show me eighteen inches. Eighteen inches are how much more than fifteen inches? Eighteen inches are how much more than one foot? What part of a foot more? In one foot how many six inches? In eighteen inches how many six inches? How many half-feet in eighteen inches? How many feet? Count eighteen inches by *sixes* of inches. (Ans. Six inches, twelve inches, eighteen inches.) What are two times six inches? Three times six inches? How many *twelves* of inches in eighteen inches? One third of eighteen inches is how many inches? Two thirds of eighteen inches?

Again, show one foot on your measures. Show eighteen inches. Eighteen inches are how much more than one foot? How many *threes* of inches in this six inches? How many *threes* of inches in the foot? Then, how many *threes* of inches in eighteen inches? Put down the measures. Let me see how many can add to eighteen inches by *threes* of inches. (Statement form: Three inches, six inches, nine inches, twelve inches, fifteen inches, eighteen inches.) Three times three inches are how many inches? Four times three inches? Five times three inches? Six times three inches? Take up the measures now. With your right hand show me exactly eighteen inches; now with your left hand show me what you think is exactly one half of eighteen inches. How many inches in the upper half? How many in the lower half? How many *nines* of inches in eighteen inches? Two times nine inches are how many inches? Nine inches and nine inches are how many inches? With your right hand show eighteen inches; with your left show fifteen inches. Three inches is what part of five inches? In fifteen inches how many *fives* of inches? In eighteen inches how many *fives* of inches?

Show on your measures sixteen inches; how many inches does this lack of being eighteen inches? Two inches are what part of four inches? How many *fours* of inches in sixteen inches? How many *fours* of inches in eighteen inches? Two inches are what part of eight inches? How many *eights* of inches in sixteen inches? How many eights of inches in eighteen inches. Show eighteen inches again. Four inches taken from eighteen leave how many inches? How many *sevens* of inches in fourteen inches? Four inches is what part of seven inches? In eighteen inches how many *sevens* of inches? Holding your measures vertically again show me the two halves of eighteen inches. How many inches in the upper? How many in the lower? Now, take one inch from the lower and add it to the upper part; how many inches in each part? Ten

inches and eight inches are how many inches? Take another inch from the lower and add it to the upper part. How many inches in each part? Eleven inches and seven inches are how many inches?

DESK WORK.

18 in. = ft. in.
18 in. = ft.
12 in. + ½ ft. = in.
18 in. ÷ 6 in. =
⅓ of 18 in. = ft.
⅔ of 18 in. = ft.
10 in. + in. = 18 in.
14 in. + in. = 1½ ft.
14 in. ÷ 7 in. =
18 in. ÷ 7 in. =
15 in. + in. = 18 in.
15 in. ÷ 5 in. =
18 in. ÷ 5 in. =
6 in. × 3 = in.
16 in. + in. = 1½ ft.
16 in. ÷ 8 in. =
18 in. ÷ 8 in. =
16 in. ÷ 4 in. =
18 in. ÷ 4 in. =
15 in. ÷ 3 in. =
18 in. ÷ 3 in. =
⅙ of 18 in. = in.
⅚ of 18 in. = in.
½ of 18 in. = in.

LESSON 42—CLASS WORK.

NOTE.—Let a number of pupils stand before the class and using foot measures, make a line by placing them end to end horizontally, just eighteen feet long, holding the measures rather low.

How long is this line of measures? Now, beginning at this end, raise this much (three feet) of the line higher. Class, how long is the part of the line just raised above the other part? In this way you may raise the whole line—three feet at a time—and *as you do so* the class may add by threes to the other end of the line. (Ans. Three feet, six feet, nine feet, twelve feet, fifteen feet, eighteen feet.) Lower the line. You may raise it again in the same manner, this time counting the *threes* of feet. (Ans. One three of feet, two *threes* of feet, three *threes* of feet, four *threes* of feet, five *threes* of feet, six *threes* of feet.)

In eighteen feet how many threes of feet? How many yards in eighteen feet? Lower the line. This time we will raise two feet at a time, counting the *twos*.

NOTE.—The illustration and answer should be given simultaneously

(Ans. One two, two twos, three twos, four twos, five twos, six twos.) In eighteen feet how many twos of feet?

NOTE.—So show the *fours*, *fives*, and *sixes*.

DESK WORK

15 ft. + 3 ft. = ft.	18 ft. ÷ 4's of ft. =
15 ft. + 1 yd. = yd.	15 ft. ÷ 5's of ft. =
5 yd. + 3 ft. = ft.	18 ft. ÷ 5's of ft. =
18 ft. ÷ 3's of ft. =	18 ft. ÷ 9's of ft. =
3 ft. × 6 = yd.	$\frac{1}{2}$ of 6 yd. = ft.
$\frac{1}{6}$ of 18 ft. = yd.	$\frac{1}{2}$ of 18 ft. = yd.
15 ft. ÷ 3's of ft. =	$\frac{1}{3}$ of 18 ft. = yd.
12 ft. ÷ 3's of ft. =	$\frac{2}{3}$ of 18 ft. = ft.
12 ft. ÷ 6's of ft. =	$\frac{5}{6}$ of 18 ft. = yd.
18 ft. ÷ 6's of ft. =	3 yd. + 9 ft. = ft.
16 ft. ÷ 4's of ft. =	$3\frac{1}{3}$ yd. + ft. = 18 ft.

LESSON 43—CLASS WORK.

NOTE.—Let four pupils stand before the class, each having a yard measure, holding it vertically.

How many yard measures do you see? Place them in a horizontal straight line. How many yards long is this line? How many feet long is the line? Four times three feet are how many feet? Henry may add another yard measure; how many yards long now, class? How many feet long? Five times three feet are how many feet? John may add another yard measure; how many yards long now? How many feet long? Six times three feet are how many feet? In eighteen feet how many *threes* of feet?

In fifteen feet (teacher removing one yard measure) there are how many yards? In sixteen feet (teacher adding one foot measure) there are how many yards? In seventeen feet (teacher adding another foot measure) there are how many yards? In fourteen feet (teacher removing a yard measure) there are how many yards? In thirteen feet (teacher removing one foot measure) there are how many yards? In twelve feet (removing another foot measure) there are how many yards?

DESK WORK.

15 ft. ÷ 3's of ft. =
16 ft. = yd. ft.
17 ft. = yd. ft.
18 ft. = yd.
18 ft. ÷ 3's of ft.
12 ft. ÷ 3's of ft.
14 ft. = yd. ft.
13 ft. = yd. ft.
3 yd. + 2 ft. = ft.

$4\frac{1}{3}$ yd. = yd. ft.
$4\frac{2}{3}$ yd. = yd. ft.
3 ft. × 3 = yd.
3 yd. × 2 = yd.
3 yd. × 2 = ft.
3 ft. × 5 = yd.
1 yd. × 5 = ft.
1 yd. × 6 = yd.
1 yd. × 6 = ft.

LESSON 44—CLASS WORK.

Make from yard measures a line of measures five yards long.

NOTE.—A number of children are called to arrange the yard measures in a continuous line before the class.

How many measures have you used? How many yards long is the line? How many feet long is it? Add another yard measure. Five yards and one yard are how many yards? Fifteen feet and one yard are how many feet?

Now children, here is a new measure; this small rope is just one *rod* long, so we shall call it our rod measure. Let us measure the "line" with the rod measure. Is the line more or less than a rod in length? Louis will use

the foot measure and find how much longer it is. The line, then, is how long? (Ans. The line is one rod and one and one-half feet long.) Then how many feet long is the *rod measure*? Sixteen and one-half feet and one and one-half feet are how many feet? Remove from the line of measures one of the yard measures. How many yards long is the line now? How many feet long? Let us measure it with the rod measure; is it more or less than a rod in length? How much more? A rod is how much more, then, than fifteen feet? One and one-half feet are what part of a yard? Fifteen feet are how many yards? Fifteen feet and one and one-half feet are how many yards? The *rod measure*, then, is how many yards long? Is sixteen feet more or less than a rod? How much less? What part of a yard less? Is seventeen feet more or less than a rod? How much more? What part of a yard more?

One rod is how many yards more than twelve feet? One rod is how much more in feet than twelve feet? What is one half of sixteen and one-half feet (one rod)? What is one half of eight and one-fourth feet?

NOTE.—Tell the children how the rod is used.

DESK WORK.

18 ft. ÷ 1-yd. =	1 rd. + 1½ ft. = yd.
18 ft. = rd. ft.	18 ft. − 1½ ft. = rd.
18 ft. = rd. yd.	16½ ft. − 1½ ft. = yd.
15 ft. + yd. = 18 ft.	16½ ft. + 1½ ft. = yd.
15 ft. + ft. = 1 rd.	½ of 18 ft. = yd.
15 ft. + yd. = 1 rd.	½ of 1 rd. = ft.
16 ft. + ft. = 1 rd.	½ of 8¼ ft. = ft.
1 rd. + ft. = 17 ft.	18 ft. − 1 rd. = ft.
5 yd. + 1½ ft. = rd.	18 ft. − 1 rd. = yd.

Lesson 45—Class Work.

Weigh one pound of salt.

Weigh one-half pound of salt.

Weigh one-fourth pound of salt.

Weigh one-eighth pound of salt.

In one pound of salt there are how many ounces? One ounce of salt is what part of a pound of salt? Weigh one sixteenth of a pound of salt. How many one-sixteenths pound of salt in one eighth pound of salt? Weigh three-sixteenths pound of salt. Weigh five-sixteenths pound of salt. Weigh seven sixteenths. Nine sixteenths. Eleven sixteenths. Thirteen sixteenths. Fifteen sixteenths. What does the last quantity lack of being a pound?

Note.—Review the class on the *parts* and *factors* of sixteen without objects.

Hold up the quantity you first weighed. Hold up another quantity which put with it would make eighteen ounces of salt. (Pupil's statement: One pound and two ounces of salt are eighteen ounces of salt.) Two ounces of salt are what part of a pound of salt? Eighteen ounces of salt are how many pounds of salt? How many one eighths of a pound of salt in eighteen ounces? How many *twos* of ounces in eighteen ounces?

Hold up fifteen ounces of salt. Show another quantity which, if put with it, would make eighteen ounces of salt. How many ounces in the last quantity? How many *threes* of ounces in fifteen ounces? In eighteen ounces? Find three bags that together contain seven eighths of a pound of salt. How many ounces does it contain? Show another bag that has enough more to make eighteen ounces. How many *fours* of ounces in eighteen ounces? Show three eighths of a pound of salt. How many ounces? Six ounces and how many ounces make eighteen ounces? How many *sixes* of ounces in eighteen ounces? How many *fives* of ounces in eighteen ounces? How many *sevens*? How

many *eighths*? How many *nines*? How many *tens*? How many *twelves*?

How many pounds in eighteen ounces? How many half pounds? How many fourth pounds? How many eighth pounds? How many sixteenth pounds?

DESK WORK.

1 lb. + 2 oz. = oz.	18 oz. ÷ 1 lb. =
$\frac{3}{4}$ lb. + oz. = $1\frac{1}{8}$ lb.	18 oz. ÷ $\frac{1}{2}$ lb. =
15 oz. + 3 oz. = lb.	18 oz. ÷ $\frac{1}{4}$ lb. =
15 oz. ÷ 5's of oz. =	18 oz. ÷ $\frac{1}{8}$ lb. =
18 oz. ÷ 5's of oz. =	8 oz. × 2 = lb.
15 oz. ÷ 3's of oz. =	9 oz. × 2 = oz.
18 oz. ÷ 3's of oz. =	6 oz. × 2 = oz.
16 oz. ÷ 8's of oz. =	6 oz. × 3 = lb.
18 oz. ÷ 8's of oz. =	2 oz. × 8 = lb.
16 oz. ÷ 4's of oz. =	2 oz. × 9 = oz.
18 oz. ÷ 4's of oz. =	18 oz. × $\frac{1}{2}$ = oz.

LESSON 46—CLASS WORK.

THE POUND AS A UNIT OF REFERENCE.

NOTE.—Write the problem, $\frac{1}{2} + \frac{1}{8} = ?$, on the board. Analysis. One half pound of salt is eight ounces (weighing a half pound of salt); one eighth of a pound of salt (weighing this quantity) is two ounces; one ounce (holding up the ounce weight) is one sixteenth of a pound of salt, eight ounces (holding up this quantity) are eight sixteenths of a pound of salt, and two ounces (showing this quantity) are two sixteenths of a pound of salt; eight sixteenths of a pound of salt and two sixteenths of a pound of salt (pouring both into one bag) are five eighths of a pound of salt. Therefore, etc.

PROBLEMS.

$\frac{1}{16} + \frac{1}{8} =$	$\frac{3}{16} \div \frac{1}{8} =$	$\frac{3}{8} \times \frac{1}{2} =$
$\frac{1}{8} - \frac{1}{16} =$	$\frac{1}{8} \div \frac{3}{16} =$	$\frac{5}{8} + \frac{9}{16} =$
$\frac{1}{8} \div \frac{1}{16} =$	$\frac{1}{2} + \frac{3}{8} =$	$\frac{5}{8} - \frac{1}{4} =$
$\frac{1}{2}$ of $\frac{1}{8} =$	$\frac{1}{2} - \frac{3}{8} =$	$\frac{3}{4} - \frac{5}{8} =$
$\frac{3}{16} + \frac{1}{8} =$	$\frac{1}{2} \div \frac{3}{8} =$	$\frac{3}{4} + \frac{7}{16} =$
$\frac{3}{16} - \frac{1}{8} =$	$\frac{3}{8} \div \frac{1}{2} =$	$\frac{3}{4} \div \frac{7}{16} =$

DRILLS FOR BLACKBOARD OR CHART.

Drill 1.

$3 = \frac{}{3}$	$8\frac{1}{2} = \frac{}{2}$	$4\frac{1}{2} = \frac{}{2}$	$2\frac{5}{6} = \frac{}{6}$
$3 = \frac{}{4}$	$4\frac{1}{4} = \frac{}{4}$	$1\frac{1}{3} = \frac{}{3}$	$5\frac{1}{2} = \frac{}{2}$
$4 = \frac{}{4}$	$5\frac{2}{3} = \frac{}{3}$	$5 = \frac{}{5}$	$4\frac{2}{3} = \frac{}{3}$
$2 = \frac{}{2}$	$6 = \frac{}{5}$	$5 = \frac{}{2}$	$9 = \frac{}{2}$
$2 = \frac{}{3}$	$2\frac{1}{2} = \frac{}{2}$	$4 = \frac{}{4}$	$4\frac{3}{4} = \frac{}{4}$
$2 = \frac{}{4}$	$1\frac{1}{2} = \frac{}{2}$	$3 = \frac{}{2}$	$2\frac{1}{8} = \frac{}{8}$
$7\frac{1}{2} = \frac{}{2}$	$3\frac{1}{3} = \frac{}{3}$	$3\frac{1}{8} = \frac{}{8}$	$2\frac{5}{7} = \frac{}{7}$

Drill 2.

$\frac{7}{3} =$	$\frac{9}{4} =$	$\frac{15}{2} =$	$\frac{19}{3} =$	$\frac{17}{3} =$	$\frac{13}{10} =$
$\frac{6}{3} =$	$\frac{7}{2} =$	$\frac{15}{4} =$	$\frac{19}{9} =$	$\frac{17}{4} =$	$\frac{17}{10} =$
$\frac{6}{2} =$	$\frac{14}{3} =$	$\frac{10}{3} =$	$\frac{10}{6} =$	$\frac{15}{6} =$	$\frac{11}{10} =$
$\frac{8}{5} =$	$\frac{16}{7} =$	$\frac{14}{2} =$	$\frac{17}{5} =$	$\frac{13}{6} =$	$\frac{19}{10} =$
$\frac{11}{6} =$	$\frac{18}{5} =$	$\frac{17}{8} =$	$\frac{15}{8} =$	$\frac{11}{10} =$	$\frac{18}{10} =$
$\frac{12}{5} =$	$\frac{18}{2} =$	$\frac{18}{7} =$	$\frac{16}{5} =$	$\frac{15}{10} =$	$\frac{16}{10} =$

Drill 3.

$\left.\begin{matrix} \frac{1}{2} \\ \frac{1}{4} \\ \frac{3}{4} \\ 2 \\ 1\frac{1}{2} \\ 1\frac{3}{4} \\ 1\frac{1}{4} \\ 2\frac{3}{8} \end{matrix}\right\} = \frac{}{8}$

Drill 4.

$\left.\begin{matrix} \frac{1}{3} \\ 1 \\ \frac{2}{3} \\ 1\frac{1}{3} \\ 1\frac{2}{3} \\ 2 \\ \frac{1}{9} \\ \frac{1}{6} \end{matrix}\right\} = \frac{}{9}$

Drill 5.

$\left.\begin{matrix} \frac{1}{2} \\ \frac{1}{5} \\ \frac{3}{5} \\ \frac{4}{5} \\ 1 \\ 1\frac{1}{5} \\ 1\frac{2}{5} \\ 1\frac{4}{5} \end{matrix}\right\} = \frac{}{10}$

Drill 6.

$\left.\begin{matrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \\ \frac{1}{6} \\ \frac{2}{3} \\ \frac{3}{4} \\ \frac{5}{6} \\ 1\frac{1}{2} \end{matrix}\right\} = \frac{}{12}$

Drill 7.

$\frac{1}{2}$ $\frac{1}{7}$ 1 $\frac{2}{7}$ $\frac{5}{7}$ $\frac{3}{7}$ $\frac{4}{7}$ $1\frac{2}{7}$ } = $\frac{}{14}$

Drill 8

$\frac{1}{2}$ 3 4 5 $\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{7}{8}$ $\frac{1}{8}$ 1 } = $\frac{}{16}$

$\frac{5}{10}$ = $\frac{4}{8}$ = $\frac{2}{4}$ = $\frac{6}{8}$ = $\frac{6}{10}$ = $\frac{3}{12}$ = $\frac{4}{6}$ = $\frac{4}{12}$ = $\frac{3}{6}$ = $\frac{2}{12}$ = $\frac{12}{16}$ = $\frac{6}{12}$ = $\frac{2}{16}$ = $\frac{9}{12}$ = $\frac{2}{6}$ = $\frac{5}{10}$ = $\frac{8}{12}$ = $\frac{2}{10}$ = $\frac{10}{12}$ = $\frac{8}{10}$ = } =

Drill 9

$\frac{2}{14}$ $\frac{1}{16}$ $\frac{4}{14}$ $\frac{10}{14}$ $\frac{10}{16}$ $\frac{8}{14}$ $\frac{8}{16}$ $\frac{6}{14}$ $\frac{6}{16}$ $\frac{12}{14}$ $\frac{12}{16}$ $\frac{3}{9}$ $\frac{14}{16}$ $\frac{6}{9}$ } =

$\frac{6}{4}$ $\frac{9}{6}$ $\frac{12}{8}$ $\frac{18}{16}$ $\frac{15}{12}$ } =

NOTE —In the following blackboard drills lead the pupils to see in the number below the line some *unit* with which they are familiar and at first to read the *name* of this unit instead of the number. Example: *3* is the number below the line while the one above is 2; the pupil then says, 2 feet are $\frac{2}{3}$ of a *yard*. Again, *4* is the number below the line; the pupil says, 2 quarts are $\frac{1}{2}$ of a *gallon*. Again, *10* is the number below the line; the pupil says, 2 cents are $\frac{1}{5}$ of a *dime*. After the class is able to do this well then let the pupils recite as follows: 2 feet are $\frac{2}{3}$ of *3 feet*. 2 quarts are $\frac{1}{2}$ of *4 quarts*, 2 cents are $\frac{1}{5}$ of *10 cents*. Finally, have them recite, making the numbers *abstract*; as, 2 are $\frac{2}{3}$ of *3*; 2 are $\frac{1}{2}$ of *4*; 2 are $\frac{1}{5}$ of *10*. Let each pupil recite *through* a drill before sitting.

1. $\frac{1}{2, 3, 4, 5, 7, 8, 10, 12, 14, 16}$

2. $\frac{2}{2, 3, 4, 5, 7, 8, 10, 12, 14, 16}$

3. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 3

4. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 4

5. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 5

6. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 6

7. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 7

8. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 8

9. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 9

10. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 10

11. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 11

12. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 12

13. 2, 3, 4, 6, 7, 8, 10, 12, 14, 16 — 13

14. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 14

15. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 15

16. 2, 3, 4, 5, 7, 8, 10, 12, 14, 16 — 16

1. $\frac{1}{6}$

$\frac{1}{4}, \frac{1}{8}, \frac{1}{2}, \frac{2}{8}, \frac{3}{4}, \frac{3}{8}, \frac{1}{16}, \frac{6}{16}, \frac{7}{16}, \frac{8}{16}, \frac{9}{16}$

2. $\frac{1}{2}$

$\frac{1}{3}, \frac{2}{3}, \frac{1}{6}, \frac{3}{6}, \frac{5}{12}, \frac{7}{12}, \frac{1}{2}, \frac{1}{5}, \frac{2}{5}, \frac{1}{10}, \frac{3}{10}, \frac{4}{8}, \frac{9}{10}$

3. $\frac{3}{8}$

$\frac{1}{4}, \frac{1}{8}, \frac{1}{2}, \frac{3}{8}, \frac{3}{4}, \frac{5}{8}, \frac{1}{16}, \frac{3}{16}, \frac{7}{16}, \frac{9}{16}$

4. $\frac{3}{4}$

$\frac{1}{4}, \frac{1}{8}, \frac{1}{2}, \frac{3}{8}, \frac{4}{4}, \frac{5}{8}, \frac{1}{16}, \frac{3}{16}, \frac{7}{16}, \frac{9}{16}$

5. $\frac{2}{5}$

$\frac{1}{2}, \frac{1}{5}, \frac{2}{5}, \frac{1}{10}, \frac{3}{10}, \frac{4}{8}, \frac{7}{10}, \frac{9}{10}$

6. $\frac{3}{7}$

$\frac{2}{7}, \frac{1}{14}, \frac{1}{7}, \frac{1}{2}, \frac{5}{14}, \frac{5}{7}, \frac{6}{7}, \frac{9}{14}, \frac{7}{14}, \frac{13}{14}$

7. Recite the factors of 12. (Ans. $12 = \begin{cases} 3 \times 4 \\ 4 \times 3 \\ 6 \times 2 \\ 2 \times 6 \end{cases}$)

Recite the factors of 14.
Recite the factors of 15.
Recite the factors of 16.
Recite the factors of 18.
Recite the factors of 10.
Recite the factors of 9.
Recite the factors of 8.

CHAPTER XV.

Nineteen—Twenty—Fractions.

NOTE 1.—Develop *nineteen* through the use of linear, liquid, and dry measure. United States money, avoirdupois weight, and time measure should also be employed. Lessons will now be easily planned after those that have been given on other numbers.

NOTE 2.—Keep up the *drills* on abstract combinations to nineteen.

NOTE 3.—Alter and extend the blackboard *drills* given in Chapter XII. Also, form similar *drills*, using the *nickel* unit and cents, the *dime* unit and cents, the *week* unit and days, the *fortnight* unit and days, and the *pound* unit and ounces.

NOTE 4.—Continue drills in "column addition", sums not to exceed nineteen.

LESSON 47—CLASS WORK.

Place on the table a gallon, two half-gallon and three pint measures. Fill them with water. How many pints of water in the gallon measure? In the two half-gallon measures? In the three measures? How many pints of water in pint measures? In all the measures? I will now add another pint of water to that already on the table; nineteen pints of water and one pint of water are how many pints of water? How many pints of water on the table? The four pints of water in pint measures are what part of a gallon of water? How many gallons of water on the table? How many *eights* of pints of water on the table? Show a *four* of pints of water in pint measures;

arrange them in vertical order. Show another *four* of pints of water. *Another.* Show two *fours* of pints of water in one quantity. How many *fours* of pints in twenty pints of water? Twenty pints of water are how many times four pints of water? Four pints are what part of a gallon? Twenty pints are how many half gallons? Show four *fours* of pints of water; how many pints? Sixteen pints of water and four pints of water are how many pints of water? Place two pint measures together and all the others in another group; how many pints of water in the larger group of measures? In the smaller? In both? Eighteen pints of water and two pints of water are how many pints of water? Change one pint measure from the larger to the smaller group; how many pints now in each? Seventeen pints of water and three pints of water are how many pints of water?

Place in one group a half-gallon and one pint measure; how many pints? Then call it a *five* of pints of water? How many pints in all the other measures? How many *fives* of pints in all the other measures? How many *fives* of pints in twenty pints? Change five pints from the larger group to the smaller; how many pints in each group? Ten pints of water and ten pints of water are how many pints of water? Two times ten pints of water are how many pints of water? How many *tens* of pints of water in twenty pints of water? Place one pint measure upon another pint measure; this is a *two* of pints of water. Two pints of water taken from twenty pints of water leave how many pints of water? In eighteen pints of water there are how many *twos* of pints of water? In twenty pints of water how many *twos* of pints of water? Eight times two pints of water are how many pints of water? Nine times two pints? Ten times two pints? How many quarts of water in twenty pints of water? In eighteen pints? In sixteen pints? In fourteen pints? In twelve pints?

How many *threes* of pints in eighteen pints? In twenty pints?

How many *sixes* of pints in eighteen pints? In twenty pints?

How many *sevens* of pints in fourteen pints? In twenty pints?

DESK WORK.

19 pt. + 1 pt. = pt.	5 pt. × 3 = gal.
20 pt. = gal. pt.	5 pt. × 4 = pt.
16 pt. + 4 pt. = gal.	10 pt. + 10 pt. = gal.
20 pt. ÷ 8's of pt. =	1¼ gal. + 1¼ gal. = pt.
20 pt. ÷ ½ gal. =	10 pt. × 2 = pt.
20 pt. ÷ 4's of pt. =	20 pt. ÷ 10's of pt. =
4 pt. × 5 = pt.	10 pt. ÷ 2's of pt. =
20 pt. − 2 pt. = pt.	20 pt. ÷ 2's of pt. =
20 pt. − 2 pt. = gal.	2 pt. × 9 = pt.
17 pt. + pt. = 2½ gal.	2 pt. × 10 = pt.
15 pt. ÷ 5's of pt. =	18 pt. ÷ 3's of pt. =
20 pt. ÷ 5's of pt. =	20 pt. ÷ 3's of pt. =

LESSON 48—CLASS WORK.

NOTE.—There will be needed for this lesson two dimes, four nickels and several one-cent pieces.

How many cents in this dime? In this one? In both What is one half of twenty cents? Two halves? How many *tens* of cents in twenty cents? How many cents in this nickel? How many nickels in this dime? In this dime? In both? How many nickels in twenty cents? How many *fives* of cents in twenty cents? What is one fourth of twenty cents? Two fourths? Three fourths? Four fourths? How many *twos* of cents in this dime? In this one? In both? How many *twos* of cents in twenty cents? What is one tenth of twenty cents? Three tenths? Seven tenths? Nine tenths?

Place in one quantity a ten, a five, and a one-cent piece; and in another four one-cent pieces. How many cents in the first quantity? In the second? Do you see a *four* of cents? How many *fours* of cents in sixteen cents? How many *fours* of cents in twenty cents? What is one fifth of twenty cents? Two fifths? Three fifths? Four fifths? Five fifths?

Eighteen cents and how many cents are twenty cents? Show these two quantities. How many *threes* of cents in eighteen cents? In twenty cents? In eighteen cents how many *sixes* of cents? In twenty cents? In eighteen cents how many *nines* of cents? In twenty cents?

Fourteen cents and how many cents are twenty cents? Show these two quantities. How many *sevens* of cents in fourteen cents? In twenty cents? Sixteen cents and how many cents are twenty cents? Show these two quantities. How many *eights* of cents in sixteen cents? In twenty cents?

Separate twenty cents into two quantities, putting eight cents in one; how many are in the other? Eight cents are what part of twelve cents? How many *twelves* of cents in twenty cents?

LESSON 49—CLASS WORK.

Weigh one pound; how many ounces? Weigh one fourth of a pound; how many ounces? Sixteen ounces and four ounces are how many ounces? How many pounds? How many *fours* of ounces in sixteen ounces? In twenty ounces? Weigh three fourths of a pound; how many ounces? Weigh one half pound; how many ounces? Eight ounces and twelve ounces are how many ounces? Are how many pounds? Weigh three fourths of a pound; and three eighths of a pound; how many ounces in each? Twelve ounces and six ounces are how many ounces?

How many more ounces would make twenty ounces? How many sixes of ounces in twelve ounces? In eighteen ounces? In twenty ounces? Weight seven eighths of a pound; how many ounces? Weigh three eighths of a pound; how many ounces? In both together how many ounces? How many *sixes* of ounces in fourteen ounces? In twenty ounces? Weigh five eighths of a pound; how many ounces? How many such quantities in twenty ounces? How many *tens* of ounces in twenty ounces? How many *fives* of ounces in ten ounces? In twenty ounces? How many *fours* of ounces in ten ounces? How many *fours* of ounces in twenty ounces? Weigh one and one eighth pounds; how many ounces? How much less than twenty ounces? How many *nines* of ounces in eighteen ounces? How many *nines* of ounces in twenty ounces?

DESK WORK.

20 oz. − 4 oz. = lb.
16 oz. ÷ 4 oz. =
20 oz. ÷ 4 oz. =
12 oz. + 8 oz. = lb. oz.
12 oz. + $\frac{3}{8}$ lb. = oz.
12 oz. ÷ 6 oz. =
18 oz. ÷ 6 oz. =
20 oz. ÷ 6 oz. =
14 oz. + 6 oz. = oz.
14 oz. ÷ 6 oz. =

20 oz. ÷ 10's of oz. =
20 oz. ÷ 5's of oz. =
20 oz. ÷ 4's of oz. =
18 oz. ÷ 9's of oz. =
20 oz. ÷ 9's of oz. =
16 oz. + $\frac{1}{4}$ lb. = oz.
$\frac{3}{4}$ lb. + $\frac{1}{2}$ lb. = oz.
20 oz. − $\frac{3}{8}$ lb. = oz.
$\frac{7}{8}$ lb. + $\frac{3}{8}$ lb. = oz.
$1\frac{1}{8}$ lb. + 2 oz. = lb.

LESSON 50—CLASS WORK.

NOTE.—Let each pupil have a yard measure. Always have measures held vertically in this kind of lesson.

With your left hand show me on the measure eighteen inches, and with your right show me twenty inches. How many inches *between* your left and your right hand?

Eighteen inches and two inches are how many inches? In eighteen inches how many *twos* of inches are there? In twenty inches how many *twos* of inches are there? Nine times two inches are how many inches? Ten times two inches are how many inches?

With your left hand show on the measure ten inches, and with the right show twenty inches. Which is the greater *length*, from the top of the measure to your left hand or from your left hand to your right hand? How many inches in each length? Ten inches and ten inches are how many inches? Two times ten inches are how many inches?

With your left hand show fifteen inches; with your right hand show twenty inches; how many inches *between* your hands? Fifteen inches and five inches are how many inches? In fifteen inches there are how many *fives* of inches? In twenty inches there are how many *fives* of inches? Four times five inches are how many inches?

With your left hand show sixteen inches; with your right show twenty inches. *Between* the points you are showing there are how many inches? Sixteen inches and four inches are how many inches? In sixteen inches there are how many *fours* of inches? In twenty inches there are how many *fours* of inches? Five times four inches are how many inches?

Use your left hand to point out eighteen inches again, and your right to show twenty inches. In eighteen inches how many *sixes* of inches? In twenty inches? In eighteen inches how many *threes* of inches? In twenty inches? Show sixteen inches and twenty inches. In sixteen inches how many *eights* of inches? In twenty inches?

Show twelve inches and twenty inches. How many feet in twenty inches?

DESK WORK.

18 in. + 2 in. = in.
$1\frac{1}{2}$ ft. + 2 in. = ft.
$1\frac{2}{3}$ ft. − $1\frac{1}{2}$ ft. = in.
18 in. ÷ 2's of in. =
20 in. ÷ 2's of in. =
2 in. × 9 = ft.
2 in. × 10 = ft.
10 in. + 10 in. = in.
10 in. × 2 = ft.
15 in. + in. = 20 in.

15 in. ÷ 5's of in. =
20 in. ÷ 5's of in. =
5 in. × 4 = ft.
16 in. + 4 in. = in.
16 in. + $\frac{1}{3}$ ft. = ft.
16 in. ÷ 4's of in. =
20 in. ÷ 4's of in. =
20 in. ÷ 3's of in. =
16 in. ÷ 8's of in. =
20 in. ÷ 8's of in. =

LESSON 51—CLASS WORK.

Here is a foot measure; what part of a yard is it? Roy may add another foot measure to this; join it to one end. Two feet are what part of a yard? Roy may get another foot measure and join it to one end of these two; three feet are what part of a yard, or how many yards? Ethel may join on another foot measure, and recite. Ethel, another, and recite. Bessie may join on another measure, and recite. Bessie, another, and recite. Hazel may join on another, and recite. Hazel, still another; recite. Carl may add another; recite. Carl, another; recite. Margaret may add one, and recite, and then another, and recite. John may add two, and recite after adding each. Mary; Byron; Villa may add one, and recite.

We will begin again with Roy; Roy, you may take your two measures from the line of measures that we have formed; how many remain? Eighteen feet are how many yards? Ethel may take away her two measures, and recite. Bessie may take away her two measures, and recite. Hazel may take away her two, and recite. Now Carl; now, Margaret; now, John, etc.

DESK WORK.

19 ft. + 1 ft. = ft.
19 ft. + 1 ft. = yd. ft.
19 ft. + 1 ft. = yd.
18 ft. + yd. = 20 ft.
6 yd. + 2 ft. = ft.
17 ft. + yd. = 20 ft.
$5\frac{2}{3}$ yd. + 3 ft. = yd.
$5\frac{2}{3}$ yd. + 3 ft. = ft.
20 ft. − $1\frac{1}{3}$ yd. = ft.
20 ft. − 4 ft. = yd.

15 ft. ÷ 1 yd. =
18 ft. ÷ 1 yd. =
16 ft. ÷ 3's of ft. =
19 ft. ÷ 3's of ft. =
14 ft. ÷ 3's of ft. =
17 ft. = yd. ft.
17 ft. = yd.
17 ft. ÷ 3's of ft. =
12 ft. ÷ 3's of ft. =
$\frac{1}{2}$ of 20 ft. = yd.

LESSON 52—CLASS WORK.

NOTE.—Let five pupils, each having a yard measure, holding it vertically, stand before the class.

How many yard measures do you see? Place them, now, in a horizontal straight line; how many yards long is this line? How many feet long is the line? Five times three feet are how many feet? Robert may add another yard measure to the line; how many yards long is the line now? How many feet long? Six times three feet are how many feet? Now, Sam may add two foot-measures to the line. How long is the line now in yards? In feet? In twenty feet there are how many yards and feet? In twenty feet there are how many yards? How many *threes* of feet? In nineteen feet (teacher removing one foot measure) there are how many yards? In eighteen feet (removing the other foot measure) there are how many yards? In seventeen feet (removing a yard measure and replacing it with two foot measures) there are how many yards? In fifteen feet? In sixteen feet? In fourteen feet? In twelve feet? In thirteen feet? etc.

NOTE.—Restore the twenty-foot line.

Twenty feet are how much more than a rod; measure. In twenty feet there are how many rods and feet? In

twenty feet there are how many rods and yards and feet?
In twenty feet there are how many rods and yards?

DESK WORK.

5 yd. × 3 = yd.	15 ft. ÷ 3's of ft. =
5 ft. × 3 = ft.	17 ft. ÷ 3's of ft. =
6 ft. × 3 = ft.	16 ft. ÷ 1 yd. =
6 yd. + ft. = 20 ft.	1 rd. + $3\frac{1}{2}$ ft. = ft.
6 yd. + 2 ft. = yd.	1 rd. + 1 yd. = ft.
19 ft. ÷ 3's of ft. =	20 ft. = rd. yd. ft.
18 ft. ÷ 1 yd. =	20 ft. = yd. ft.

A TEST LESSON

In twenty inches there are how many feet?
In twenty feet there are how many yards?
In twenty feet there are how many rods?
In twenty feet there are how many fathoms?
In twenty square feet there are how many square yards?
In twenty cents there are how many dimes?
In twenty cents there are how many nickels?
In twenty quarts there are how many gallons?
In twenty pints there are how many gallons?
In twenty pints there are how many quarts?
In twenty pecks there are how many bushels?
In twenty quarts there are how many pecks?
In twenty ounces there are how many pounds?
In twenty things there are how many dozen?
In twenty things there are how many half dozen?
In twenty days there are how many weeks?
In twenty days there are how many fortnights?
In twenty weeks there are how many fortnights?
In five bushels how many pecks?
In two dimes how many cents?
In one and one fourth pounds how many ounces?
In one and one half feet how many inches?
In six and one third yards how many feet?
In two and one half gallons how many pints?

DESK WORK.

20 in. = ft. in.	20 oz. ÷ 1 lb. =
20 in. ÷ 1 ft. =	20 things ÷ 1 doz. =
20 ft. ÷ 1 yd. =	20 da. ÷ 1 wk. =
20 ft. ÷ 1 fathom =	20 da. ÷ 14 da. =
20 sq. ft. ÷ 1 sq. yd. =	20 wk. ÷ 1 fortnight =
20 ct. ÷ 1 dime =	5 bu. ÷ 1 pk. =
20 ct. ÷ 5 ct. =	2 di. ÷ 1 ct.
20 qt. ÷ 1 gal. =	$1\frac{1}{4}$ lb. ÷ 1 oz. =
20 pt. ÷ 1 gal. =	$1\frac{1}{2}$ ft. ÷ 1's of in. =
20 pt. ÷ 1 qt. =	$6\frac{1}{3}$ yd. ÷ 1's of ft. =
20 pk. ÷ 1 bu. =	$2\frac{1}{2}$ gal. ÷ 1's of pt. =

LESSON 53—CLASS WORK.

NOTE.—After *twenty* has been thoroughly taught, then the *ream* may be introduced. Teach the ream as a *unit* and show its division into quires *objectively*. Let the first step be *thorough* and *exact*. Number is easy enough if the foundation for it is properly laid. Next, teach thoroughly the *parts* of twenty by reference to the *ream* as a *unit*. For instance, one half of a ream of paper is —— quires; one fourth of a ream of paper is —— quires; one twentieth of a ream of paper is —— quire. Two quires are what part of a ream? Four quires are what part of a ream? etc.

THE REAM AS A UNIT OF REFERENCE.

NOTE.—Let the problem $\frac{1}{4}+\frac{1}{5}=?$ be written on the board. Analysis: One fourth of a ream of paper is five quires; one fifth of a ream of paper is four quires; one quire of paper is one twentieth of a ream; five quires are five twentieths of a ream, and four quires are four twentieths of a ream; five twentieths of a ream of paper and four twentieths of a ream of paper are nine twentieths of a ream of paper. Therefore, $\frac{1}{4}+\frac{1}{5}=\frac{9}{20}$.

NOTE.—The foregoing analysis may be readily adapted to the following problems:—

$\frac{1}{2}+\frac{1}{5}=?$	$\frac{3}{20}-\frac{1}{10}=?$	$\frac{1}{4}\div\frac{4}{5}=?$
$\frac{1}{4}+\frac{2}{5}=?$	$\frac{9}{10}-\frac{1}{2}=?$	$\frac{1}{2}\div\frac{3}{4}=?$
$\frac{3}{4}+\frac{3}{20}=?$	$\frac{4}{5}-\frac{3}{4}=?$	$\frac{1}{2}$ of $\frac{1}{10}=?$
$\frac{7}{10}+\frac{1}{20}=?$	$\frac{2}{5}-\frac{7}{20}=?$	$\frac{1}{3}$ of $\frac{3}{4}=?$

$\frac{13}{20} + \frac{1}{10} = ?$	$\frac{1}{2} \div \frac{1}{5} = ?$	$\frac{3}{5}$ of $\frac{3}{4} = ?$
$\frac{1}{2} + \frac{3}{10} = ?$	$\frac{1}{4} \div \frac{1}{5} = ?$	$\frac{2}{3}$ of $\frac{9}{10} = ?$
$\frac{4}{5} + \frac{1}{10} = ?$	$\frac{1}{10} \div \frac{1}{20} = ?$	$\frac{1}{4}$ of $\frac{2}{5} = ?$
$\frac{1}{2} - \frac{3}{20} = ?$	$\frac{1}{20} \div \frac{1}{10} = ?$	$\frac{3}{10} \times \frac{1}{2} = ?$
$\frac{4}{5} - \frac{1}{10} = ?$	$\frac{1}{5} \div \frac{1}{4} = ?$	$\frac{1}{5} \times \frac{1}{2} = ?$
$\frac{3}{4} - \frac{2}{5} = ?$	$\frac{3}{4} \div \frac{1}{10} = ?$	$\frac{1}{4} \times \frac{2}{5} = ?$
$\frac{1}{4} - \frac{1}{5} = ?$	$\frac{4}{5} \div \frac{1}{4} = ?$	$\frac{11}{20} - \frac{1}{5} = ?$

SUGGESTIONS.

1. As the work progresses the teacher will find the *units* given in the following table of great value. The intelligent teacher will now be able to make the proper use of them. The units are the pillars on which the whole structure rests, and hence their function and value cannot be too well understood:—

20 { shillings = 1 pound
pennyweights = 1 ounce
grains = 1 scruple
days = 1 school month
hundred weight = 1 ton
quires = 1 ream
things = 1 score

24 { hours = 1 day
sheets = 1 quire
grains = 1 pennyweight

25 cents = 1 quarter
27 cu. ft. = 1 cu. yd.
28 days = 1 lunar month
30 days = 1 [illegible]
32 quarts = 1 bushel
36 inches = 1 yard

2. Those who have followed the course thus far will find no difficulty in forming lessons in both class and desk work for the study of numbers to 37. The new units of measure given in the table may, at the proper time, be introduced both for integers and fraction work. The use of linear, liquid, dry, money, time, and avoirdupois measure should continue *with actual measuring*, though *empty* measures may be used to a considerable extent for illustrative purposes in the treatment of numbers from 20 to 37.

3. The *1-inch square* should be used in the *first* development of numbers from 37 to 50. Practice and desk work should continue to involve all the kinds of measure the pupils have had.

NOTE.—For the *1-inch square* as a primary unit of measure, see Chapter XVI. and Chapter XVIII.

4. Drill on the combinations.—

5	3	3	3	4	5	4	5	6	6	6	7	8	6
4	2	4	3	4	2	1	3	6	5	3	1	2	4
7	7	6	9	8	9	8	7	8	9	8	9	8	7
7	3	2	8	7	6	4	5	8	4	6	5	5	6

5 Give ample drill in column addition to 50.

Add:—

1.	2.	3.	4.	5.	6.
6	5	4	3	6	2
4	4	3	6	2	6
5	6	5	4	1	3
1	3	2	5	4	5
3	1	6	2	5	4
6	2	1	4	3	1
2	6	0	6	2	2
5	5	3	3	6	3
3	3	2	5	5	6
4	4	5	1	2	5

Add:—

1.	2.	3.	4.	5.	6.	7.	8.
8	5	6	7	4	3	9	2
3	2	9	8	7	8	2	8
5	7	7	9	5	2	8	3
6	8	2	6	6	7	3	6
4	6	5	5	3	4	7	9
7	4	8	3	9	9	4	4
9	3	4	2	8	6	6	5
2	9	3	6	2	5	5	7
6	6	6	4	6	6	6	6

Add:—

1.	2.	3.	4.	5.	6.
9	1	5	2	8	7
8	8	3	5	6	6
7	7	9	8	1	7
7	9	7	6	4	8
8	5	6	7	7	0
6	4	8	9	3	9

6. Drill on the multiplication table to 50.

MULTIPLICATION TABLE.
(4 TO 50)

2 times 2 = 4

2 times 3
3 times 2 } = 6

2 times 4
4 times 2 } = 8

3 times 3 = 9

2 times 5
5 times 2 } = 10

2 times 6
6 times 2
3 times 4
4 times 3 } = 12

2 times 7
7 times 2 } = 14

3 times 5
5 times 3 } = 15

2 times 8
8 times 2
4 times 4 } = 16

2 times 9
9 times 2
3 times 6
6 times 3 } = 18

2 times 10
10 times 2
4 times 5
5 times 4 } = 20

2 times 11
11 times 2 } = 22

2 times 12
12 times 2
3 times 8
8 times 3
4 times 6
6 times 4 } = 24

5 times 5 = 25

3 times 9
9 times 3 } = 27

4 times 7
7 times 4 } = 28

3 times 10
10 times 3
6 times 5
5 times 6 } = 30

4 times 8
8 times 4 } = 32

3 times 11
11 times 3 } = 33

5 times 7
7 times 5 } = 35

3 times 12
12 times 3
4 times 9
9 times 4
6 times 6 } = 36

4 times 10
10 times 4
5 times 8
8 times 5 } = 40

6 times 7
7 times 6 } = 42

4 times 11
11 times 4 } = 44

5 times 9
9 times 5 } = 45

4 times 12
12 times 4
6 times 8
8 times 6 } = 48

7 times 7 = 49

5 times 10
10 times 5 } = 50

PART THIRD.

CHAPTER XVI.

Fifty-four—Fractions.

OUTLINE FOR THIRD YEAR.

First month, teach to sixty.

Second month, teach to seventy.

Third month, teach to eighty-five.

Fourth month, teach to one hundred.

Fifth month, teach Reading and Writing Numbers with Addition.

Sixth month, teach Subtraction and Multiplication and continue the work of the fifth month.

Seventh month, teach Division and continue the work of the last two months.

Eighth month, teach Division, and continue the work of the last three months.

Ninth month, Review.

NOTE.—Work in fractions is to be kept up *throughout the year.*

NOTE.—Begin percentage after numbers have been taught to 100 and continue it to the end of the year.

SUGGESTIONS.

1. Make a *thorough review* of Part Second. Do not take up new work until this is done. Cover all features of the work. In this review it will not be necessary to use objects—measures and material—to any great extent. Pupils should be able to recite with *facility* and *accuracy* any work passed over. This review should include:—

(*a*) *All factors* of all products to 50.

(*b*) *Explanations* of combinations and comparisons of fractional quantities, bringing into use all the *Units of Reference* to 50.

(*c*) *Fractional parts* of all units of measure to 50.

(*d*) *Recitation* of all *drills* extended to 50.

(*e*) The "*Combinations*".

(*f*) *Column Addition.*

2. All work once passed over should be kept fresh in the minds of the pupils.

Lesson 1—Class Work.

Note.—Teach the following:—

1. A 1-inch square is a square one inch long and one inch wide.
2. A 2-inch square is a square two inches long and two inches wide.
3. A 3-inch square is one three inches long and three inches wide.

Draw on your slates a 1-inch square.

Draw on your slates a 2-inch square.

Draw on your slates a 3-inch square.

What is a 4-inch square? Draw a 4-inch square. What is a 5-inch square? Draw a 5-inch square. What is a 6-inch square? Draw a 6-inch square. What is a 7-inch square? Draw one.

How many 1-inch squares in a 2-inch square?

How many 1-inch squares in a 3-inch square?

How many 1-inch squares in a 5-inch square?

How many 1-inch squares in a 4-inch square?

How many 1-inch squares in a 7-inch square?

How many 1-inch squares in a 6-inch square?

What are the two equal factors of 9?

What are the two equal factors of 16?

What are the two equal factors of 36?

What are the two equal factors of 25?

What are the two equal factors of 49?

What is the square of 3?
What is the square of 5?
What is the square of 7?
What is the square of 4?
What is the square of 6?

LESSON 2—CLASS WORK.

Draw a 2-inch square and a 4-inch square.

A 4-inch square is how many times a 2-inch square?

A 2-inch square is what part of a 4-inch square?

Draw a 6-inch square in place of the 4-inch square. The 2-inch square is what part of the 6-inch square? The 6-inch square is how many times the 2-inch square? Draw a 7-inch square in place of the 6-inch square. The 7-inch square is how many times the 2-inch square? Draw a 3-inch square and a 6-inch square. The 3-inch square is what part of the 6-inch square? The 6-inch square is how many times the 3-inch square?

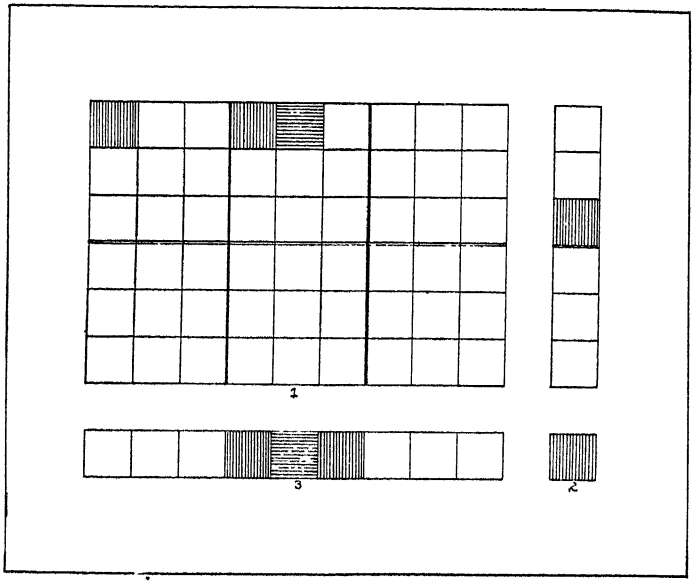
NOTE.—Make other comparisons. Pupils should use measures in drawing the squares. This work should be reviewed when the number 64 (the next square above 49) has been reached, and again on 81 and on 100.

DESK WORK.

25 ct. ÷ 5-ct. =
49 sq. ft. ÷ 9-sq. ft. =
49 pk. ÷ 4-pk. =
49 oz. ÷ 16-oz. =
49 qt. ÷ 4-qt. =
36 qt. ÷ 4-qt. =
36 pk. ÷ 4-pk. =
36 sq. ft. ÷ 9-sq. ft. =
36 oz. ÷ 16-oz. =

25 qt. ÷ 4-qt. =
25 pk. ÷ 4-pk. =
25 sq. ft. ÷ 9-sq. ft. =
16 qt. ÷ 4-qt. =
16 pk. ÷ 4-pk. =
16 sq. ft. ÷ 9-sq. ft. =
9 pk. ÷ 4-pk. =
9 qt. ÷ 4-qt. =
49 ct. ÷ 10 ct. =

THE INCH SQUARE AS A PRIMARY UNIT OF MEASURE.



1. Diagram; figure; rectangle.
2. An inch-square; primary unit.
3. A measure; a *nine* of inch-squares.
4. A measure, a *six* of inch-squares

EXPLANATION AND USE OF THE DIAGRAM.

In the development of numbers a new unit of measure is now introduced. The aim of the next lesson is to show the factors of 54 through the use of the diagram above and by a true process of measurement. By combinations of inch-squares the measures 3 and 4 are derived.

The number 54 is selected on which to base the lesson because well adapted to the purpose. The work begins with 50, but it will be easy after studying the diagram (with explanations) to *adapt* it to any number and to form suitable desk lessons for the children.

The figure, or set of figures, is to be drawn upon the blackboard; it should stand for several days for purposes of review. It contains 54 1-inch squares, but this fact is *to be discovered* by the pupils led, of course, by the teacher.

NOTE.—Carefully study the following.—

1. The primary unit is an inch-square, and the figure below the principal one and that to its right, measures. The measures are also named from the number of inch-squares they contain; as, a *nine* of inch squares, a *six* of inch squares.

2. It will be observed that some of the squares in the measures are shaded. The object of this is to separate them into groups, so that the number of squares in the measure may be ascertained *without counting*. To prepare the class for the addition by groups the teacher should rapidly *point out* the different groups requiring the pupils to name the number of squares (calling them inch-squares) in each group.

NOTE.—Do this several times at first, in irregular order, then (directing the class to add) point out the groups *to be added*. To secure alertness and promptness the groups should always be pointed out irregularly, i. e. "skipping around", whether the pupils are *to name* the number of inch-squares in the group or *to add*. The number of inch-squares in the measure (3) having been determined, the pupils may call it a *nine* of inch-squares. Measure (4) is a *six* of inch-squares.

3. It will be observed that the upper row of squares is separated into groups. This is because the number 54, which is a new number to the class, is to be reached toward the end of the process *by additions*. 54 being a new (unknown) number and 53 an old (known) one the final step would be to get 54 by the combination of 53 and 1, hence the *one* shaded square in the upper left corner of the diagram. However, at this stage of advancement 54 might easily be derived from other combinations; as, 52 and 2 or 51 and 3 or 50 and 4. All that is necessary to

get this variation is to *change the order* in adding the groups in the upper row to the whole number of squares in the figure below the upper row. The number of squares exclusive of the upper row is found by irregular multiplications, as will be seen more clearly in the lesson on the development of 54 following this explanation.

4. It has now been ascertained that the figure contains 54 inch-squares. The next step is to lead the pupils *to discover* the fact that: In 54 1-inch squares there are six *nines* of 1-inch squares. Also, that: In 54 1-inch squares there are nine *sixes* of 1-inch squares. Also, that: In 54 1-inch squares there are two *twenty-sevens* of 1-inch squares. Also, that: In 54 1-inch squares there are three *eighteens* of 1-inch squares. The first of these facts is illustrated in the rectangle by the horizontal rows of squares; the second, by the vertical rows; the third, by the horizontal sections separated by the double line drawn through the center of the diagram, and the fourth, by the vertical sections formed by the heavy vertical lines. It may be advisable to draw *one* of each of these sections in a separate figure as each becomes in the process a *measure* and is to be compared with the *whole* and the whole with it.

5. A sort of trowel-shaped *cover* about 14 inches long, 9 inches wide, and $\frac{1}{4}$ of an inch thick, made of a smooth, light, well-seasoned board with a handle at the center of one side, will aid materially in the presentation of the work. In fact, something of the kind is indispensable. Its use is to show that portion of the rectangle involved in the question immediately before the class *by covering* the part not so involved.

6. The inch-square as a primary unit in development may be conveniently introduced on *about* the number 40. It is, therefore, used in both second and third grade work. This work forms a very suitable introduction to square measure, which comes later on.

LESSON 3—CLASS WORK.

Pupils, I will point out the groups of 1-inch squares in the figure on the blackboard; as I do so I want you, together, to *name* the number of inch-squares in each group. Now, pay very close attention and see how quick you can be—not to make mistakes. (Ans. 2 inch-squares; 3 inch-squares; 1 inch-square; 3 inch-squares; 3 inch-squares; 4 inch-squares; 2 inch-squares; 4 inch-squares.) Now, you may *add* as I point out the groups in this measure. (Ans. 3 inch-squares; 6 inch-squares; 9 inch-squares.) Now, in this measure. *Add.* (Ans. 3 inch-squares; 4 inch-squares; 6 inch-squares.) What then is the first measure? (Ans. The first measure is a *nine* of inch-squares.) What is the other measure? (Ans. It is a *six* of inch-squares.)

Now, Mary may come to the board and recite as I show *with the cover* what I want. (Teacher shows the two lower rows of figure 1.) (Ans. In 18 inch-squares there are 2 *nines* of inch-squares; in 36 inch-squares there are 4 *nines* of inch-squares; in 27 inch-squares there are 3 *nines* of inch-squares; in 45 inch-squares there are 5 *nines* of inch-squares.)

Class add. (Ans. 45 inch-squares (teacher pointing out the groups in the upper row), 49 inch-squares, 51 inch-squares, 53 inch-squares, 54 inch-squares.) Then, in the entire rectangle, how many 1-inch squares? In the entire rectangle how many *nines* of inch-squares?

Willie may come and recite. (Teacher covers all the rectangle except the two vertical rows at the right.) (Ans. In 12 inch-squares, there are 2 *sixes* of inch-squares; in 24 inch-squares, there are 4 *sixes* of inch-squares; in 18 inch-squares, there are 3 *sixes* of inch-squares; in 36 inch-squares, there are 6 *sixes* of inch-squares; in 30 inch-squares, there are 5 *sixes* of inch-squares; in 48 inch-squares, there are 8 *sixes* of inch-squares; in 42 inch-

squares, there are 7 *sixes* of inch-squares; in 54 inch-squares, there are 9 *sixes* of inch-squares.) Nine times six inch-squares are how many inch-squares? Six times nine inch-squares are how many inch-squares? In the section below the double line how many *nines* of inch-squares are there? How many inch-squares in this section? Three times *nine* inch-squares are how many inch-squares? How many such sections in the rectangle? In 54 inch-squares, how many *twenty-sevens* of inch-squares? In the section to the right of the first heavy vertical line, how many sixes of inch-squares? Three times *six* inch-squares are how many inch-squares? How many such sections in the rectangle? Three times *eighteen* inch-squares are how many inch-squares? In 54 inch-squares how many *eighteens* of inch-squares?

DESK WORK.

54 sq. ft. ÷ 9-sq. ft. =
54 sq. ft. ÷ 1-sq. yd. =
9 sq. ft. × 5 = sq. ft.
9 sq. ft. × 5 = sq. yd.
9 sq. ft. ÷ 1 sq. yd. =
9 sq. ft. × 2 = sq. yd.
36 sq. ft. ÷ 9 sq. ft. =
36 sq. ft. ÷ 1 sq. yd. =
9 sq. ft. × 3 = sq. yd.
6 ft. = fathom.

9 f. = ft.
9 ft. × 6 = f.
9 ft. × 6 = ft.
48 ft. ÷ 1-f. =
6 ft. × 6 = f.
27 cu. ft. = cu. yd.
27 cu. ft. × 2 = cu. yd.
36 cu. ft. = cu. yd.
54 cu. ft. ÷ 1 cu. yd. =
54 ft. ÷ 1-f. =

LESSON 4—CLASS WORK.

NOTE.—Review the first part of Lesson 3.

9 inch-squares (using the *cover*) are what part of 54 inch-squares?

What are $\frac{2}{6}$ of 54 inch-squares?
What are $\frac{4}{6}$ of 54 inch-squares?
What are $\frac{5}{6}$ of 54 inch-squares?
What are $\frac{3}{6}$ of 54 inch-squares?

What is $\frac{1}{9}$ of 54 inch-squares?
What are $\frac{2}{9}$ of 54 inch-squares?
What are $\frac{5}{9}$ of 54 inch-squares?
What are $\frac{3}{9}$ of 54 inch-squares?
What are $\frac{7}{9}$ of 54 inch-squares?
What are $\frac{8}{9}$ of 54 inch-squares?
What are $\frac{9}{9}$ of 54 inch-squares?
What is $\frac{1}{3}$ of 54 inch-squares?
What are $\frac{2}{3}$ of 54 inch-squares?
What is $\frac{1}{2}$ of 54 inch-squares?
What are $\frac{2}{2}$ of 54 inch-squares?
What are $\frac{3}{3}$ of 54 inch-squares?
What are the factors of 54?

(Answer: $54 = \begin{cases} 9 \times 6 \\ 6 \times 9 \\ 18 \times 3 \\ 3 \times 18 \\ 27 \times 2 \\ 2 \times 27 \end{cases}$)

What are the factors of 52?
What are the factors of 51?
What are the factors of 50?
What are the factors of 49?
What are the factors of 48?
What are the factors of 46?
What are the factors of 45?

Desk Work.

50 ct. + ct. = 54 ct.
54 ct. = dimes ct.
54 ct. = d.
54 ct. = nickels ct.
54 ct. = n.
54 ct. = d. n.
54 ct. − 5 ct. = ct.
48 ct. + ct. = 54 ct.
54 ct. − 3 ct. = ct.
44 ct. + 10 ct. = ct.
44 ct. + 9 ct. = ct.
54 ct. − 27 ct. = d.
54 ct. − 27 ct. = n.
18 ct. × 2 = ct.
36 ct. + 18 = ct.
27 ct. × 2 = d.

LESSON 5—CLASS WORK.

NOTE.—Review development of 54, Lesson 3.

NOTE.—The following desk lessons should be given after such preparation for them by similar oral questions and by such illustrative work *with the measures* as may be found necessary:

DESK WORK.

52 pt. + pt. = 54 pt.
54 pt. − 4 pt. = pt.
54 pt. − 4 pt. = qt.
54 pt. ÷ 2 pt. =
54 pt. ÷ 1 qt. =
54 pt. − 3 qt. = qt.
54 pt. − 3 qt. = gal.
54 pt. = gal. pt.
54 pt. = gal.
54 pt. = qt.
27 pt. × 2 = qt.

18 pt. × 3 = gal.
$\frac{1}{3}$ of 54 pt. = qt.
$\frac{2}{9}$ of 54 pt. = gal.
$\frac{5}{9}$ of 54 pt. = qt.
$\frac{1}{6}$ of 54 pt. = gal.
$\frac{1}{3}$ of 54 pt. = qt.
$\frac{5}{6}$ of 54 pt. = pt.
$\frac{2}{3}$ of 54 pt. = qt.
$\frac{7}{9}$ of 54 pt. = pt. = qt
$\frac{8}{9}$ of 54 pt. = gal.
$\frac{1}{3}$ of 54 pt. = gal.

DESK WORK.

48 in. ÷ 1 ft. =
54 in. = ft. in.
54 in. = ft.
18 ft. × 3 = yd.
27 ft. × 2 = yd.
17 yd. + ft. = 54 ft.
36 in. + 1 ft. = yd.

36 in. + 1 ft. + in. = 54 in.
54 ft. ÷ 1 yd. =
54 in. − 27 in. = ft. in.
54 in. − 18 in. = yd.
$\frac{1}{2}$ of 54 ft. = yd.
$\frac{2}{3}$ of 54 in. = ft.
$\frac{1}{9}$ of 54 ft. = yd.

DESK WORK.

1 bu. 2 pk. + qt. = 54 qt.
1 pk. × 6 = qt.
54 pt. = pk. qt.
54 qt. = pk.
54 qt. = bu. qt.
54 qt. = bu.

18 pk. × 3 = bu.
$\frac{1}{3}$ of 54 qt. = bu.
$\frac{2}{3}$ of 54 qt. = pk.
$\frac{1}{9}$ of 54 pk. = qt.
$\frac{2}{9}$ of 54 bu. = pk.
$\frac{5}{6}$ of 54 qt. = qt.

Desk Work.

1 lb. × 3 = oz. 18 oz. × 2 = lb.

48 oz. + oz. = 54 oz. $\frac{1}{9}$ of 27 lb. = oz.

54 oz. ÷ 1 lb. = $\frac{5}{6}$ of 54 oz. = oz.

18 oz. × 2 = lb. oz. $\frac{1}{3}$ of 54 oz. = lb.

54 da. = wk. da. 54 da. = wk.

54 da. = fort's. wk. da. 54 da. = fortnights.

54 quires = reams quires.

54 hr. = da. hr. 54 da. = mon. da.

Desk Work.

Note.—Prepare the pupils for the following desk work. This will lead up to the quick mastery of the "fundamental rules". When the children can solve these little problems, several such questions should be given every day for practice.

1.	3) 27	5. 54 − 18	9. 27 × 2	13. 3) 54
2.	6) 36	6. 54 − 27	10. 26 × 2	14. 2) 54
3.	6) 48	7. 54 − 4	11. 15 × 3	15. 3) 51
4.	2) 52	8. 54 − 3	12. 18 × 3	16. 3) 48

Lesson 6—Class Work.

Note.—Before taking up the work using the cubic yard as a unit of reference in fractional work impart a complete and thorough concept of the *cubic yard*. Perhaps the only feasible method of doing this is to construct a cubic foot, or rather the outline of a cubic foot, by using twelve foot measures to represent the twelve edges of the cube, and then, in like manner, to build a cubic yard by

similarly using twelve yard measures. The pupils may be easily led to discover that there are 27 cubic feet in a cubic yard. To make this fact clear the *cubic-inch blocks may be called cubic feet, and enough of them used to construct a supposed cubic yard.* Be careful to see that the *solid* nature of the cubic yard and cubic foot is apprehended. These suggestions should be worked out a number of times in the class to avoid the possibility of vagueness. Do not trust to mere word descriptions of that which is vitally important when you can do better. *Drill* on the *parts* of the cubic yard.

THE CUBIC YARD AS A UNIT OF REFERENCE.

NOTE.—Write the problem, $\frac{5}{9} + \frac{10}{27} =$ what? on the board. Explanation: Five ninths of a cubic yard are fifteen cubic feet; one cubic foot is one twenty-seventh of a cubic yard, and fifteen cubic feet are fifteen twenty-sevenths of a cubic yard; fifteen twenty-sevenths of a cubic yard and ten twenty-sevenths of a cubic yard are twenty-five twenty-sevenths of a cubic yard. Therefore, $\frac{5}{9} + \frac{10}{27} = \frac{25}{27}$.

NOTE.—In like manner, teach such fractions as the following:—

$\frac{5}{9} - \frac{10}{27} =$	$\frac{2}{3} + \frac{11}{27} =$	$\frac{2}{3} + \frac{7}{9} =$
$1 - \frac{10}{27} =$	$\frac{3}{3} - \frac{11}{27} =$	$\frac{3}{3} - \frac{7}{9} =$
$1 - \frac{5}{27} =$	$\frac{2}{3} \div \frac{11}{27} =$	$\frac{2}{3} \div \frac{7}{9} =$
$\frac{5}{9} \div \frac{10}{27} =$	$\frac{11}{27} \div \frac{2}{3} =$	$\frac{7}{9} \div \frac{2}{3} =$
$\frac{10}{27} \div \frac{5}{9} =$	$\frac{2}{3} \times \frac{1}{2} =$	$\frac{7}{9} \times 3 =$
$\frac{5}{9} \times \frac{1}{3} =$	$\frac{2}{3} \times \frac{1}{9} =$	$\frac{7}{9} \times \frac{1}{3} =$

CHAPTER XVII.

Sixty—Fractions.

SUGGESTIONS.

The teacher should have a light, high table for the work with the cubes. The top need not be more than sixteen

inches square. It should be light so that it can be easily *lifted* and *turned*; this is frequently necessary during the recitation to show sections and sides of the solid that cannot otherwise be seen well from the front. It should be high enough to enable the teacher *to stand erect* while conducting the recitation, and to allow the class sitting *to see*.

It is neither practicable nor necessary to *treat* all the numbers from 50 to 100 in these lessons. The teacher will discover the *plan* of work and apply it to those not treated. At this stage of advancement *prime numbers* need very little attention. The *products* of the "multiplication table" are the numbers requiring thorough work.

Toward the close of each lesson, drill on the *factors* of numbers as far down from the number in hand as time will permit.

Lesson 7—Class Work.

Note.—Teach:—

1. What a 1-inch cube is; use the object.
2. What a 2-inch cube is.
3. What a 3-inch cube is.

Note.—Compare each with the other two.

Note.—When 64 has been reached, teach the 4-inch cube, then review and compare each with all the others. A box of inch cubes consisting of a hundred or more will be needed for this and subsequent work.

The inch cube a primary unit of measure.

The 1-inch cube is now to be introduced as the "primary unit" of measure in development. The suggestions and explanations on the use of the 1-inch square as the primary unit in this kind of work, in a general way, apply also to the use of the 1-inch cube. The analogy between the two processes of work is very strong, so one should be quite helpful to the other. The first step is the *construction of a solid.*

The *measure* which may be placed on the table, either in a vertical or a horizontal position, consists of the number of cubes to be placed in one of the *front rows*, and is *used in constructing the solid.* By having five or six *colored cubes*, the measure, no matter of what number of cubes it consists, may be separated into groups that the groups may be combined *to avoid counting.*

The teacher builds the solid to the required magnitude by placing *rows* or *groups* of cubes, each one *equal to* or *less* than the measure. The row or group of cubes *to be placed* is compared first with the measure. The pupils recite, of course, as the "building up" goes on. When pupils once understand what is expected, few questions will be necessary. As the teacher *builds* a pupil *recites.*

Suppose it is desired to construct a solid containing 56 1-inch cubes. It may be formed so that a front row (one facing the class) will contain seven cubes, an end row, four cubes, and a vertical row (one showing the height), two cubes.

The measure will contain seven cubes and should be called by the pupils a *seven* of 1-inch cubes. The teacher begins the construction by placing toward the front of the table (next the class) a row of *seven* or *less* 1-inch cubes after first holding it beside the measure that the pupils may determine how many cubes it contains. Suppose the row *to be placed* contains four cubes. A pupil being designated to recite will say:

"In four 1-inch cubes, there are four sevenths of a *seven* of 1-inch cubes." The row will be *completed* by the teacher's taking up and placing a group of three cubes. A pupil will say:

"In 7 1-inch cubes there *is* one *seven* of 1-inch cubes." Now, the teacher may hold beside the measure and place a group of seven cubes. A pupil recites:

"In 14 1-inch cubes, there are two *sevens* of 1-inch cubes." The last row should be placed *upon* the first and the next addition to the solid in *front* of this, building toward the class, so every change may be easily perceived. Thus, the process is continued until the solid of 56 cubes is completed. It is to be observed that the pupil does not recite until after the group has been *placed* by the teacher in the solid being constructed.

The ultimate aim of the instruction so far has been to show that, in 56 there are eight sevens. The teacher may now *move out* one of the *end sections* consisting of eight cubes and easily elicit the fact that, in 56 inch-cubes, there are seven *eights* of inch-cubes. She may next *move out* the front or back section and get the fact that, in 56 1-inch cubes there are four *fourteens* of 1-inch cubes. Then, by placing a side section upon a side section and another side section upon the remaining one, it will be easily discovered that, in 56 1-inch cubes there are two *twenty-eights* of 1-inch cubes.

The pupils have, through a true process of measurement, been led to see all the factors of the number 56. Each question leading up to these facts is an easy, natural, and interesting arithmetical problem, bringing into use and repetition numbers and processes with which the pupils are already acquainted but with which they need to become more familiar through practice.

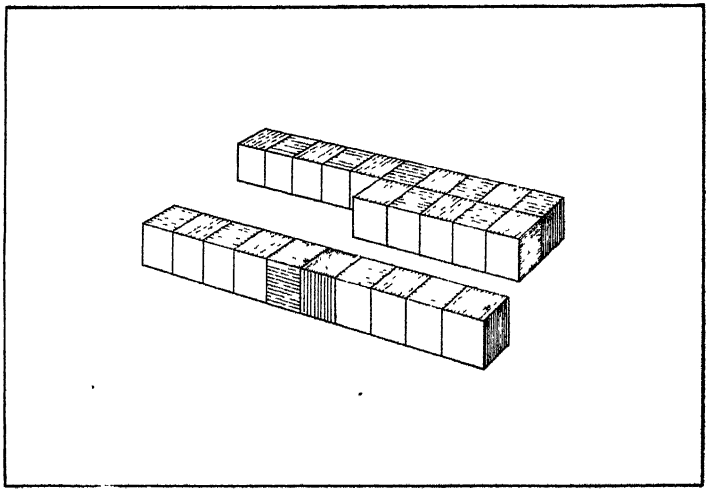
Further illustrating the plan just presented, 60 is made the basis of the following lesson.

Lesson 8—Class Work.

Add the groups in the measure. What is the measure? (Ans. The measure is a *ten* of 1-inch cubes.) I will now *build* the solid, and as I do so, Henry may recite.

"In 10 1-inch cubes, there is *one ten* of inch cubes."

"In 15 1-inch cubes, there are *one* and *one half tens* of inch cubes."



"In 20 1-inch cubes, there are *two tens* of inch cubes."

"In 23 1-inch cubes, there are *two* and *three tenths tens* of inch cubes."

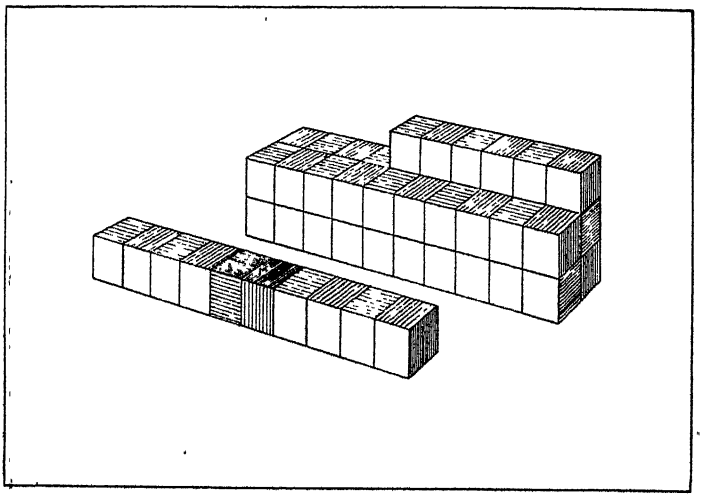
"In 30 1-inch cubes, there are *three tens* of inch cubes."

Susan may recite.

"In 34 1-inch cubes, there are *three* and *two fifths tens* of inch cubes."

"In 40 1-inch cubes, there are *four tens* of inch cubes."

"In 46 1-inch cubes there are *four* and *three fifths tens* of inch cubes."



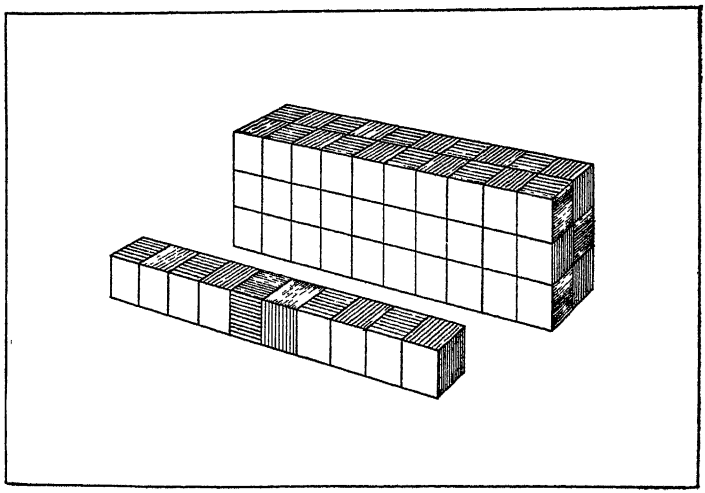
"In 50 1-inch cubes, there are *five tens* of inch cubes."

"In 58 1-inch cubes, there are *five* and *four fifths tens* of inch cubes."

DESK WORK.

60 ct. ÷ 10 ct. =	50 ct. ÷ 5 ct. =
55 ct. ÷ 10 ct. =	55 ct. ÷ 5 ct. =
40 ct. ÷ 10 ct. =	48 ct. ÷ 5 ct. =
58 ct. ÷ 10 ct. =	40 ct. ÷ 5 ct. =
46 ct. ÷ 10 ct. =	45 ct. ÷ 5 ct. =
30 ct. ÷ 10 ct. =	35 ct. ÷ 5 ct. =
32 ct. ÷ 10 ct. =	39 ct. ÷ 5 ct. =
60 ct. ÷ 5 ct. =	60 ct. ÷ 25 ct. =

"In 60 1-inch cubes, there are *six tens* of inch cubes."



The solid is now complete; how many 1-inch cubes does it contain? How many *tens* of 1-inch cubes does it contain?

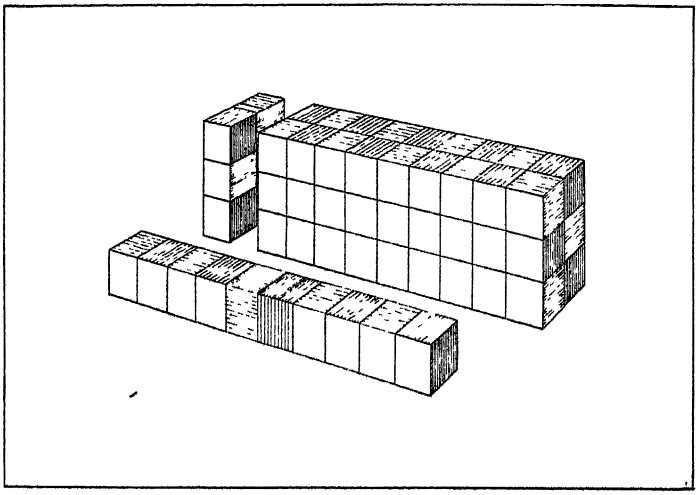
DESK WORK.

NOTE.—Review first part of lesson 8.

60 ft. ÷ 1 yd. =
17 ft. × 3 = yd.
60 ft. − 3 yd. = ft.
30 ft. + 10 yd. = ft.
18 ft. × 3 = ft.
60 ft. − 18 yd. = yd.
60 in. ÷ 1 ft. =
54 in. ÷ 1 ft. =

48 in. ÷ 12 in. =
12 in. × 3 = yd.
12 in. × $3\frac{1}{2}$ = yd.
60 ft. ÷ 1 fathom =
1 f. × 9 = ft.
60 ft. − 3 rd. = ft.
3 rd. + 3 yd. + ft. = 60 ft.
60 ft. = rd. yd.

In this section (teacher *moves out* an end section) there are how many 1-inch cubes? (Ans. There are six 1-inch cubes.)



How many such sections in the entire solid? (Ans. There are ten such sections in the entire solid.) Then how many *sixes* of 1-inch cubes in the entire solid?

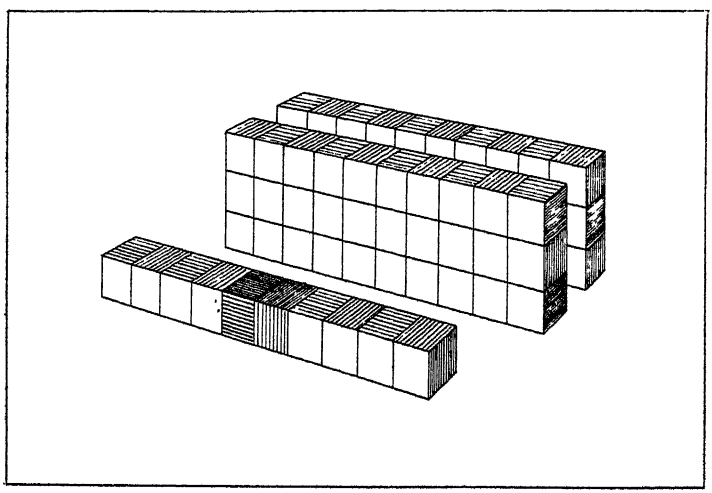
DESK WORK.

NOTE.—Review first part of lesson 8.

8 qt. × 7 = qt.	60 qt. = bu. pk. qt.
56 qt. + qt. = 60 qt.	60 qt. − 1 bu. = pk.
60 qt. ÷ 1 pk. =	15 pk. × 4 = qt.
7 pk. + 4 qt. = qt.	7 pk. × 8 = qt.
1 lb. × 3 = oz.	60 oz. ÷ 1 lb. =
60 oz. − 48 oz. = lb.	20 oz. × 3 = oz.

In the upper row of this section (teacher *moves out* a side section) there are how many 1-inch cubes? (Ans. Ten.) How many such rows in the section? (Ans. Three.) Then, in the section, there are how many 1-inch cubes? (Ans. In the section; there are 30 1-inch cubes.)

How many such sections in the solid? Then, in 60 1-inch cubes, there are how many *thirties* of inch cubes?

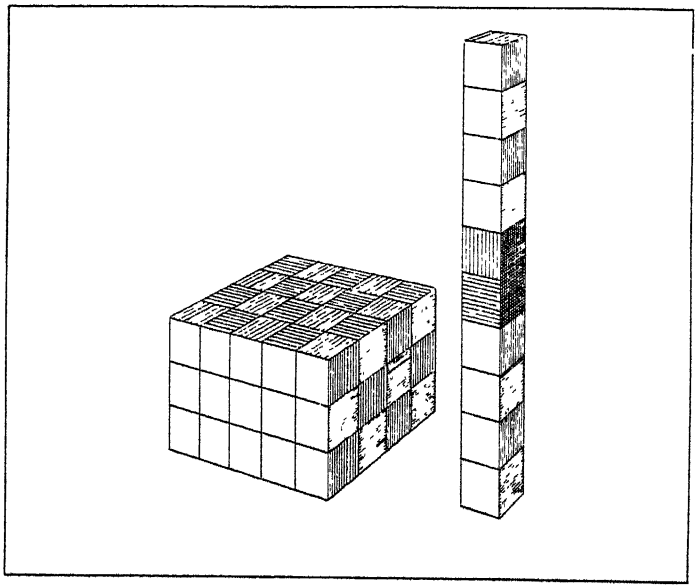


In one of these sections (showing an end row) there are how many *threes* of inch cubes? (Ans. Ten.) In both sections, how many *threes* of inch cubes? Then, in 60 1-inch cubes, there are how many *threes* of inch cubes?

DESK WORK.

1.	2.	3.	4.
15	30	17	13
15	28	18	14
15		19	16
15			10

NOTE.—Separate the solid into two equal parts, at right angles to its length, and place one half in front of the other.

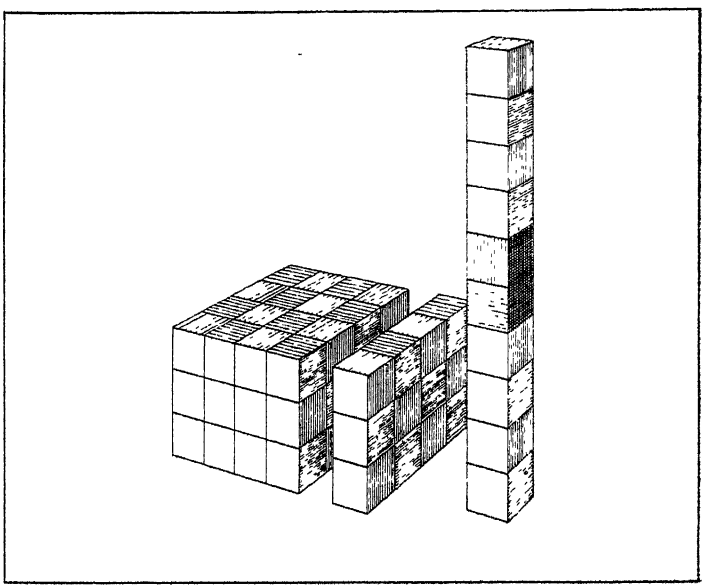


How many 1-inch cubes in the upper *row* of the front section? (Ans. Five.) How many such *rows* in the front section? How many 1-inch cubes in the front section? How many such sections in the solid? Then, how many *fifteens* of inch cubes in 60 inch-cubes?

DESK WORK.

1. 60 −30	2. 60 −50	3. 60 −32	4. 60 −48
5. 60 −56	6. 19 ×3	7. 18 ×3	8. 15 ×4

How many 1-inch cubes (moving out an end section) in the *upper row* of this section?



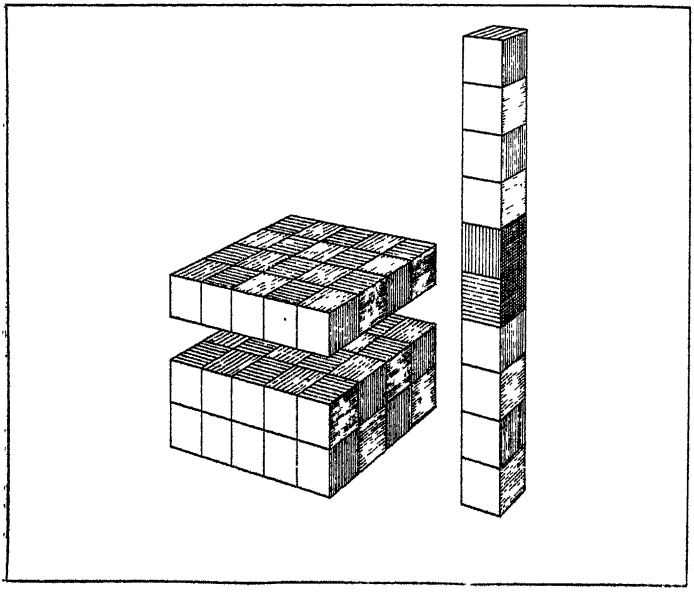
How many rows? How many 1-inch cubes in the section? How many *twelves* of inch cubes in the section? How many *twelves* of inch cubes in 60 inch-cubes?

DESK WORK

1. 29×2 2. $2)\underline{60}$ 3. $3)\underline{60}$ 4. $4)\underline{60}$

5. $5)60$ 6. $6)60$ 7. 17×3

In the front row of the top section (lifting the top section) there are how many 1-inch cubes? How many such rows in the top section?



How many 1-inch cubes in the top section? How many such sections in the solid? In 60 1-inch cubes, how many *twenties* of inch cubes?

How many *fours* of inch cubes in the top section? In the three such sections? In 60 1-inch cubes there are how many *fours* of inch cubes?

How many *fives* of inch cubes in the top section? In the three sections? In 60 1-inch cubes, there are how many *fives* of inch cubes?

Lesson 9—Class Work.

Note —Make such use of the cubes as is necessary to illustrate and teach the parts of 60

What is $\frac{1}{2}$ of 60 1-inch cubes?
What are $\frac{2}{2}$ of 60 1-inch cubes?
What is $\frac{1}{3}$ of 60 1-inch cubes?
What are $\frac{2}{3}$ of 60 1-inch cubes?
What is $\frac{1}{4}$ of 60 1-inch cubes?
What are $\frac{2}{4}$ of 60 1-inch cubes?
What are $\frac{3}{4}$ of 60 1-inch cubes?
What is $\frac{1}{5}$ of 60 1-inch cubes?
What are $\frac{2}{5}$ of 60 1-inch cubes?
What are $\frac{3}{5}$ of 60 1-inch cubes?
What are $\frac{4}{5}$ of 60 1-inch cubes?
What is $\frac{1}{6}$ of 60 1-inch cubes?
What are $\frac{2}{6}$ of 60 1-inch cubes?
What are $\frac{3}{6}$ of 60 1-inch cubes?
What are $\frac{5}{6}$ of 60 1-inch cubes?
What is $\frac{1}{10}$ of 60 1-inch cubes?
What are $\frac{3}{10}$ of 60 1-inch cubes?
What are $\frac{7}{10}$ of 60 1-inch cubes?
What are $\frac{9}{10}$ of 60 1-inch cubes?
What is $\frac{1}{12}$ of 60 1-inch cubes?
What are $\frac{5}{12}$ of 60 1-inch cubes?
What are $\frac{7}{12}$ of 60 1-inch cubes?
What are $\frac{11}{12}$ of 60 1-inch cubes?
What is $\frac{1}{15}$ of 60 1-inch cubes?
What are $\frac{2}{15}$ of 60 1-inch cubes?
What are $\frac{4}{15}$ of 60 1-inch cubes?
What are $\frac{8}{15}$ of 60 1-inch cubes?
What are $\frac{7}{15}$ of 60 1-inch cubes?
What are $\frac{8}{15}$ of 60 1-inch cubes?
What are $\frac{11}{15}$ of 60 1-inch cubes?
What are $\frac{13}{15}$ of 60 1-inch cubes?
What are $\frac{14}{15}$ of 60 1-inch cubes?

What is $\frac{1}{20}$ of 60 1-inch cubes?
What are $\frac{3}{20}$ of 60 1-inch cubes?
What are $\frac{7}{20}$ of 60 1-inch cubes?
What are $\frac{9}{20}$ of 60 1-inch cubes?
What is $\frac{1}{30}$ of 60 1-inch cubes?
What are $\frac{7}{30}$ of 60 1-inch cubes?
What are $\frac{9}{30}$ of 60 1-inch cubes?
What are $\frac{11}{30}$ of 60 1-inch cubes?
What are $\frac{25}{30}$ of 60 1-inch cubes?
What are $\frac{27}{30}$ of 60 1-inch cubes?
What are $\frac{29}{30}$ of 60 1-inch cubes?
What are $\frac{26}{30}$ of 60 1-inch cubes?
Compare $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{6}{12}$, and $\frac{5}{10}$ of 60 1-inch cubes.
Compare $\frac{1}{3}$, $\frac{3}{8}$, $\frac{5}{15}$, and $\frac{10}{30}$ of 60 1-inch cubes?

Desk Work.

$\frac{1}{2}$ of 60 da. = mon.	$\frac{13}{15}$ of 60 pt. = qt.
$\frac{1}{3}$ of 60 da. × 3 = mon.	$\frac{9}{10}$ of 60 ct. = dimes
60 hr. = da. hr.	60 ÷ 30 =
60 hr. ÷ 1 da. =	60 ÷ 20 =
60 in. ÷ 1 ft. =	60 ÷ 3 =
60 ft. ÷ 1 yd. =	60 ÷ 6 =
$\frac{19}{20}$ of 60 ft. = yd.	60 ÷ 15 =
$\frac{23}{30}$ of 60 pt. = qt.	60 ÷ 10 =
$\frac{14}{15}$ of 60 pk. = bu.	60 ÷ 4 =
$\frac{17}{20}$ of 60 ft. = yd.	60 ÷ 2 =
$\frac{7}{15}$ of 60 qt. = pt.	60 ÷ 5 =
$\frac{2}{15}$ of 60 wk. = da.	60 ÷ 12 =
$\frac{7}{12}$ of 60 da. = wk.	

LESSON 10—CLASS WORK.

THE MONTH AS A UNIT OF REFERENCE.

NOTE —Write the problem, $\frac{2}{3} + \frac{9}{10} =$ what? on the board Explanation. Two thirds of a month are twenty days, nine tenths of a month are twenty-seven days; one day is one-thirtieth of a month; twenty days are twenty-thirtieths of a month, and twenty-seven days are twenty-seven thirtieths of a month, *twenty* thirtieths of a month and *twenty-seven* thirtieths of a month are *forty-seven* thirtieths of a month, or *one* and *seventeen* thirtieths months Therefore, $\frac{2}{3} + \frac{9}{10} = 1\frac{17}{30}$ So teach the following —

$\frac{2}{3} + \frac{9}{10} =$	$\frac{1}{2} + \frac{4}{15} =$	$\frac{1}{5} + \frac{1}{6} =$
$\frac{9}{10} - \frac{2}{3} =$	$\frac{1}{2} - \frac{4}{15} =$	$\frac{1}{5} - \frac{1}{6} =$
$\frac{9}{10} \div \frac{2}{3} =$	$\frac{1}{2} \div \frac{4}{15} =$	$\frac{1}{5} \div \frac{1}{6} =$
$\frac{2}{3} \div \frac{9}{10} =$	$\frac{4}{15} \div \frac{1}{2} =$	$\frac{1}{6} \div \frac{1}{5} =$
$\frac{2}{3} \times 2 =$	$\frac{1}{2} + 1\frac{11}{30} =$	$\frac{5}{6} - \frac{4}{5} =$
$\frac{2}{3} \times \frac{1}{2} =$	$\frac{4}{15} \times 4 =$	$\frac{5}{6} + \frac{4}{5} =$

NOTE —Such lists of fractions should be left on the board for drill

CHAPTER XVIII.

Square Measure—Seventy-two—Fractions.

LESSON 11—CLASS WORK.

How many faces has this cube? What is meant by the face of a cube? What is meant by the face of the earth? What is meant by the surface of the earth? What is meant by the surface of a cube? If I say, the board is floating on the *surface* of the water, what do I mean by the word surface? Did you ever look into a stream or spring of clear water? Could you see the bottom? Could you see how deep the water was? Did you ever look at a stream or spring of *very muddy* water? Could you see the bottom? Could you see how deep it was? How far toward the bottom could you see? Do we write on the surface of the blackboard? Has the blackboard length? Show me the length. Has it width? Show me the width. Has it surface? Show me the surface. Is the surface the same as the length or width? With what is the length measured? With what is the width measured? Are length and width the same? (Ans. They are both distance or length.) Could you measure surface with the yard measure or the foot measure? Could you measure it with any of the measures you know? Do we need a measure to measure surface? Here is a piece of pasteboard; it is just one foot long and one foot wide. How much of the surface of the blackboard will it *cover*? Why? May it, then, be used to measure the surface of the blackboard? Sup-

pose I should place it *just enough times* to cover the board all over, what would that show?

NOTE.—Perform this or a similar experiment.

What other measure might I use to measure the *surface* of the blackboard? What would be a suitable measure for a very small surface?

How would you find the number of square inches in a surface? (Ans. Measure it with a square inch measure.) How would you find the number of square feet in any surface? How would you find the number of square yards in any surface?

Draw (lay off) a square inch of surface on your slates. I want each of you to bring me to-morrow sixty square-inch measures. You may make them of pasteboard. Make them exactly right.

LESSON 12—CLASS WORK.

NOTE.—Provide each pupil with a piece of paper seven inches long and five inches wide. The children need not know the dimensions

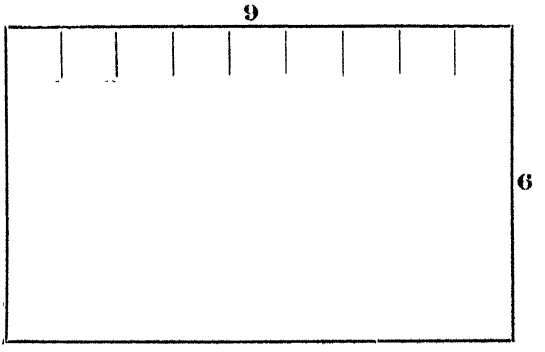
Now, place this piece of paper on your desk and use enough of the pasteboard measures to cover the surface of it. I want each one to find out for himself how many it will take. Have you finished? How many did it take? How much surface does one measure cover? (Ans. One square inch of surface.) Then, they all cover how many square inches of surface? Then, one side of the paper contains how many square inches of surface? How many measures in the first row lengthwise? How many such rows are there? Then, how many measures in all?

SUGGESTION.—Require the pupils to measure in this way the surface of several different objects. Rectangles may be drawn on their slates and measured. Their readers, copy-books or drawing-books may furnish suitable problems. Be careful to select objects whose length and width will correspond exactly with a number of

whole measures Work to the following analysis. If a surface is seven inches long and *one* inch wide, it will contain seven square inches, if it is five inches wide it will contain five times seven square inches, which is thirty-five square inches

NOTE.—It cannot be expected that one or two presentations of this subject will be sufficient It will require a few minutes each lesson for several days to make the points presented in this and the preceding lesson clear. Vary the questions and illustrations Do not crowd the subject upon the class but make it a *feature* of the work for the remainder of the year

NOTE —Place rectangles upon the board in this form and work to the following explanation:—



Explanation: If a rectangle is nine inches long and *one* inch wide it will contain nine square inches; if it is six inches wide it will contain *six* times nine square inches, which are fifty-four square inches.

Find the surface of these rectangles:—

One 8 inches long and 7 inches wide.

One 7 inches long and 7 inches wide.

One 8 inches long and 6 inches wide.

One 12 inches long and 7 inches wide.

One 12 inches long and 6 inches wide.

One 10 inches long and 7 inches wide.
One 9 inches long and 9 inches wide.
One 10 inches long and 8 inches wide.
One 9 inches long and 8 inches wide.
One 9 inches long and 7 inches wide.
One 11 inches long and 8 inches wide.
One 8 inches long and 8 inches wide.

SUGGESTIVE PROBLEMS.

1. A board is 13 inches long and 5 inches wide; what is the surface of one of its sides?

2. A brick is 8 inches long, 4 inches wide, and 2 inches thick. How much surface on the top? How much on the bottom? How much in one of the sides? In both sides? How much in one of the ends? In both ends?

3. What will be the cost of a piece of wire screen 12 feet long and 3 feet wide, at 2 cents a square foot?

4. A section of stone sidewalk is 11 feet long and 6 feet wide; how many square feet in its top surface?

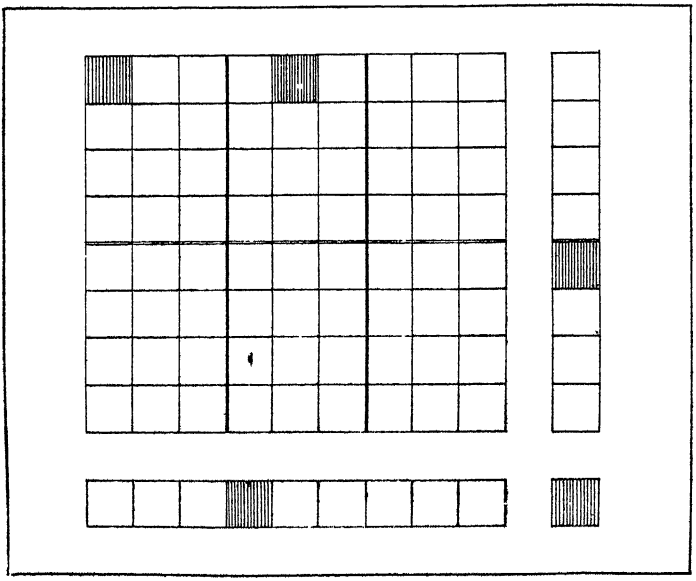
5. What is the surface of a flower-bed 9 feet long and 6 feet wide?

6. The ceiling of this room is 10 yards long and 9 yards wide. How many square yards of plastering in it?

7. The blackboard is 30 feet long and 4 feet high. How many square feet of slating does it contain?

8. How much floor space in a hall 24 yards long and 3 yards wide?

Lesson 13—Class Work.



Tell how many (pointing out the groups in the measures and in the upper row of the rectangle) inch squares in each group? What is the measure below the rectangle? (Ans. A *nine* of inch squares.) What is the measure to the right of the rectangle? (Ans. An *eight* of inch squares.) How many *nines* of 1-inch squares in 18 1-inch squares? How many *nines* of 1-inch squares in 36 inch squares? How many *nines* of 1-inch squares in 45 1-inch squares? How many *nines* of 1-inch squares in 54 1-inch squares? How many *nines* of 1-inch squares in 63 1-inch squares? Add (pointing out the groups in the upper row). (Ans. 63, 67, 70, 71, 72). In the rectangle there are how many 1-inch squares? In 72 1-inch squares, then, there are how many *nines* of 1-inch squares? Eight times *nine* 1-inch squares are how many 1-inch squares? Seven times *nine*

1-inch squares? Six times *nine* 1-inch squares? Five times *nine* 1-inch squares? What is the measure to the right? In 72 1-inch squares how many *eights* of 1-inch squares? In 70 1-inch squares how many *eights* of 1-inch squares? In 68 1-inch squares? In 64 1-inch squares? In 56 1-inch squares? In 60 1-inch squares? In 48 1-inch squares? In 50 1-inch squares? In 49 1-inch squares? In 40 1-inch squares? In 44 1-inch squares? In 32 1-inch squares?

How many *eights* of inch squares in the section to the right of the first *heavy vertical* line? How many inch squares? How many such sections in the rectangle? In 72 1-inch squares how many *twenty-fours* of inch squares? Three times twenty-four inch squares are how many inch-squares? What is one third of 72 inch squares? Two thirds?

How many inch squares in the bottom row of the rectangle? How many such rows below the double line? How many such squares in the section below the double line? How many such sections in the rectangle? In 72 inch squares there are how many *thirty-sixes* of inch squares? Two times thirty-six inch squares are how many inch squares? One half of 72 inch squares are how many inch squares?

In 36 inch squares there are how many *twelves* of 1-inch squares? In 72 1-inch squares there are how many *twelves* of inch squares? Six times twelve inch squares are how many inch squares? Twelve times six inch squares are how many inch squares?

What is one ninth of 72 1-inch squares?
What are five ninths of 72 1-inch squares?
What is one eighth of 72 1-inch squares?
What are five eighths of 72 1-inch squares?
What are the factors of 72?

DESK WORK

72 pt. ÷ 2's of pt. =	58 pt. ÷ 1 qt. =
72 pt. ÷ 1 qt. =	56 pt. ÷ 2's of pt. =
70 pt. ÷ 2's of pt. =	56 pt. ÷ 1 qt. =
70 pt. ÷ 1 qt. =	54 pt. ÷ 2's of pt. =
68 pt. ÷ 2's of pt. =	54 pt. ÷ 1 qt. =
68 pt. ÷ 1 qt. =	52 pt. ÷ 2's of pt. =
66 pt. ÷ 2's of pt. =	52 pt. ÷ 1 qt. =
66 pt. ÷ 1 qt. =	50 pt. ÷ 2's of pt. =
64 pt. ÷ 2's of pt. =	50 pt. ÷ 1 qt. =
64 pt. ÷ 1 qt. =	72 in. ÷ 36's of in. =
62 pt. ÷ 2's of pt. =	72 in. ÷ 1 yd. =
62 pt. ÷ 1 qt. =	48 in. ÷ 1 yd. =
60 pt. ÷ 2's of pt. =	60 in. ÷ 1 yd. =
60 pt. ÷ 1 qt. =	70 in. ÷ 1 yd. =

DESK WORK.

72 ft. ÷ 3's of ft. =	33 ft. × 2 = yd.
72 ft. ÷ 1 yd. =	34 ft. × 2 = yd.
36 ft. × 2 = yd.	27 ft. × 2 = yd.
35 ft. × 2 = yd.	24 ft. × 2 = yd.
36 ft. × 2 = yd.	26 ft. × 2 = yd.

DESK WORK.

72 qt. ÷ 4's of qt. =	31 qt. × 2 = gal.
72 qt. ÷ 1 gal. =	34 qt. × 2 = gal.
70 qt. ÷ 1 gal. =	29 qt. × 2 = gal.
68 qt. ÷ 4's of qt. =	56 qt. ÷ 4's of qt. =
32 qt. × 2 = gal.	56 qt. ÷ 1 gal. =
22 qt. × 3 = gal.	26 qt. × 2 = gal.
20 qt. × 3 = gal.	24 qt. × 3 = gal.

NOTE.—Form other desk lessons involving the factors 6, 8, 9, and 12.

DESK WORK.

1. 36 $\times 2$	4. 22 $\times 3$	7. 3)72	10. 2)72	13. 36	16. 72	
2. 24 $\times 3$	5. 21 $\times 3$	8. 6)72	11. 8)72	14. 72 -36	17. 72 -35	
3. 35 $\times 2$	6. 33 $\times 2$	9. 4)72	12. 9)72	15. 72 -34	18. 72 -31	

THE BUSHEL AS A UNIT OF REFERENCE.

NOTE.—Write the problem, $\frac{1}{2} + \frac{21}{32} =$ what? on the board. Explanation. One half of a bushel is sixteen quarts, one quart is *one* thirty-second of a bushel, sixteen quarts are *sixteen* thirty-seconds of a bushel, sixteen thirty-seconds of a bushel and twenty-one thirty-seconds of a bushel are thirty-seven thirty-seconds of a bushel or one and five thirty-seconds bushels. Therefore $\frac{1}{2} + \frac{21}{32} = 1\frac{5}{32}$.

NOTE.—Arrange such questions as the following for explanation and drill.—

$\frac{1}{2} + \frac{5}{32} =$	$\frac{15}{16} + \frac{3}{4} =$	$\frac{13}{16} + \frac{5}{8} =$
$\frac{1}{2} - \frac{5}{32} =$	$\frac{15}{16} - \frac{1}{4} =$	$\frac{13}{16} - \frac{5}{8} =$
$\frac{1}{2} \div \frac{5}{32} =$	$\frac{15}{16} \div \frac{3}{4} =$	$\frac{13}{16} \div \frac{5}{8} =$
$\frac{25}{32} - \frac{1}{2} =$	$\frac{3}{4} \div \frac{15}{16} =$	$\frac{5}{8} \div \frac{13}{16} =$
$\frac{1}{2} + \frac{9}{16} =$	$\frac{1}{2} + \frac{3}{8} =$	$\frac{5}{8} \times 3 =$
$\frac{1}{2} - \frac{9}{16} =$	$\frac{1}{2} \div \frac{3}{8} =$	$\frac{5}{8} \times \frac{1}{2} =$

LESSON 14—CLASS WORK.

THE PARTS OF (56) A BUSHEL OF RYE.

How many pounds in a bushel of rye?
How many pounds in one-half bushel of rye?
How many pounds in a peck of rye?
How many pounds in a half peck of rye?
How many pounds in a quart of rye?
$\frac{1}{4}$ of a bushel of rye is how many pounds?
$\frac{3}{4}$ of a bushel of rye are how many pounds?

$\frac{3}{4}$ of a bushel of rye are how many pounds?
$\frac{1}{8}$ of a bushel of rye is how many pounds?
$\frac{3}{8}$ of a bushel of rye are how many pounds?
$\frac{4}{8}$ of a bushel of rye are how many pounds?
$\frac{5}{8}$ of a bushel of rye are how many pounds?
$\frac{7}{8}$ of a bushel of rye are how many pounds?
$\frac{1}{7}$ of a bushel of rye is how many pounds?
$\frac{2}{7}$ of a bushel of rye are how many pounds?
$\frac{5}{7}$ of a bushel of rye are how many pounds?
$\frac{4}{7}$ of a bushel of rye are how many pounds?
$\frac{6}{7}$ of a bushel of rye are how many pounds?
$\frac{1}{14}$ of a bushel of rye is how many pounds?
$\frac{7}{14}$ of a bushel of rye are how many pounds?
$\frac{5}{14}$ of a bushel of rye are how many pounds?
$\frac{13}{14}$ of a bushel of rye are how many pounds?
$\frac{1}{28}$ of a bushel of rye is how many pounds?
$\frac{5}{28}$ of a bushel of rye are how many pounds?
$\frac{28}{28}$ of a bushel of rye are how many pounds?
$\frac{27}{28}$ of a bushel of rye are how many pounds?
$\frac{25}{28}$ of a bushel of rye are how many pounds?
$\frac{20}{28}$ of a bushel of rye are how many pounds?

LESSON 15—CLASS WORK.

THE BUSHEL OF RYE AS A UNIT OF REFERENCE.

NOTE.—Write on the board the problem, $\frac{1}{7}+\frac{1}{8}=$ what? Require the following explanation: One seventh of a bushel of rye is eight pounds; one eighth of a bushel of rye is seven pounds; one pound is one fifty-sixth of a bushel of rye; eight pounds are *eight* fifty-sixths of a bushel of rye, and *seven* pounds are *seven* fifty-sixths of a bushel of rye; *seven* fifty-sixths and *eight* fifty-sixths are *fifteen* fifty-sixths of a bushel of rye. Therefore, $\frac{1}{7}+\frac{1}{8}=\frac{15}{56}$.

$\frac{1}{7}+\frac{1}{8}=$	$\frac{3}{7}+\frac{3}{8}=$	$\frac{5}{28}+\frac{5}{8}=$
$\frac{1}{7}-\frac{1}{8}=$	$\frac{3}{7}-\frac{3}{8}=$	$\frac{5}{8}-\frac{5}{28}=$
$\frac{1}{7}\div\frac{1}{8}=$	$\frac{3}{7}\div\frac{3}{8}=$	$\frac{5}{8}\div\frac{5}{28}=$
$\frac{1}{8}\div\frac{1}{7}=$	$\frac{3}{8}\div\frac{3}{7}=$	$\frac{5}{28}\div\frac{5}{8}=$
$\frac{1}{7}\times\frac{1}{2}=$	$\frac{3}{7}\times\frac{1}{3}=$	$\frac{5}{8}\times\frac{1}{7}=$

NOTE.—From the following table, showing the number of pounds to the bushel in Missouri and Illinois, the teacher may select suitable *units* for teaching other numbers:—

	Wheat.	Corn.	Oats.	Barley.	Buck-wheat.	Rye.	Clover Seed.	Timothy Seed.
Missouri	60	56	35	48	52	56	60	45
Illinois	60	52	32	48	40	54	60	45

inches does the solid contain? What is the unit of measure? I will now make a solid of eight 1-inch cubes; how many cubic inches does it contain? Now, one of four inch cubes; how many cubic inches does it contain? This one has sixteen 1-inch cubes in it; how many cubic inches of wood does it contain? What is the measuring unit? By measuring the solid with the cubic inch as a measuring unit, what is found? (Ans. The solid contents are found.)

SUGGESTIONS.

1. Review the last lesson. Vary the questions and illustrations. These lessons on square and cubic measure are intended to be suggestive, representing several months' work, and are not to be taken up and taught *at the rate of a lesson a day*. The different problems or different classes of problems represent different stages of progress, and therefore the mistake of attempting the easier and the more difficult in the same lesson should be carefully avoided. The solids measured *should be actually constructed*, the pupil using the inch cubes as he gives his explanation of the problem. This should be continued *until the class is thorough in the analysis.*

2. Provide a dozen or more *rectangular solids* of different dimensions on which to base problems in the measurement of both *surfaces* and *solids*. It is important that the *object should be in sight*. A good way to get these is to ask each child that can do so to furnish *one*, being careful to give definite instructions as to the size and form of each.

3. Many such questions as the following should be given, *working to* the analysis accompanying it:—

Find the solid contents of a solid 4 inches long, 3 inches wide, and 2 inches high (thick). Analysis: A solid 1 inch long, 1 inch wide, and 1 inch high contains 1 cubic inch. If it is 4 inches long (placing 4 inch-cubes in a row), it contains 4 times 1 cubic inch, which are 4 cubic inches. If it is 3 inches wide (placing the cubes), it contains 3 times 4 cubic inches, which are 12 cubic inches. If it is 2 inches high (placing the cubes), it contains 2 times 12 cubic inches, which are 24 cubic inches. Therefore etc.

Lesson 17—Class Work.

Note.—Find the contents of these solids, always requiring the same *order* and *form* of explanation:—

One 5 inches long, 4 inches wide, and 3 inches high.
One 5 inches long, 3 inches wide, and 2 inches high.
One 6 inches long, 3 inches wide, and 2 inches high.
One 7 inches long, 3 inches wide, and 2 inches high.
One 8 inches long, 3 inches wide, and 2 inches high.
One 9 inches long, 3 inches wide, and 2 inches high.
One 10 inches long, 3 inches wide, and 2 inches high.
One 12 inches long, 3 inches wide, and 2 inches high.
One 11 inches long, 3 inches wide, and 2 inches high.
One 5 inches long, 2 inches wide, and 3 inches high.
One 5 inches long, 2 inches wide, and 4 inches high.
One 5 inches long, 2 inches wide, and 5 inches high.
One 5 inches long, 2 inches wide, and 6 inches high.
One 5 inches long, 2 inches wide, and 7 inches high.

Note.—When the class is well able to do so, it may abandon the inch cubes and use instead the rectangular solids suggested above. With these the *nature* and *form* of solids will become more prominent. They may be talked of as wood, stone, lead, etc. Place a solid before the class, state the problem, and require the explanation given above Do not hurry this work but make it a *feature* of the daily recitation.

Suggestive Problems.

1. How many cubic feet in a stone 3 feet long, 2 feet wide, and 2 feet high?

2. A vat 6 feet long, 5 feet wide, and 3 feet deep will hold how many cubic feet of water?

3. A mortar box is 8 feet long, 4 feet wide, and 3 feet deep. How many cubic feet of mortar will it contain?

4. The bed of a sand wagon is 9 feet long, 4 feet wide, and $2\frac{1}{2}$ feet deep. How many cubic feet of sand will it hold? How many cubic yards?

5. A gas chamber 5 feet long, 4 feet wide, and 4 feet high will contain how many cubic feet of gas?

6. This schoolroom is 10 yards long, 6 yards wide, and 4 yards high. How many cubic yards of air will fill it?

7. My sleeping-room is 5 yards long, 4 yards wide, and 3 yards high. How many cubic yards of space does it contain?

8. How many cubic yards of earth must be taken out to make a cellar 4 yards long, 4 yards wide, and 3 yards deep?

9. What is the shape of a block of ice 3 feet long, 3 feet wide, and 3 feet thick? How many cubic feet does it contain? How many cubic yards?

10. What is the shape of a block of marble 4 feet long, 4 feet wide, and 4 feet thick? How many cubic feet in it? How many cubic yards?

11. A cubic yard of earth is considered a *load*. How many loads of dirt will it take to fill a ditch 14 yards long, 3 yards wide, and 2 yards deep?

12. How many cubic yards of gravel will it take to make a walk 20 feet long, 4 feet wide, and 1 foot deep?

LESSON 18—CLASS WORK.

I shall now build a new solid and I wish you to see how many new things you can learn from it. First let us examine the measure. How many 1-inch cubes in this group? And in this group? How many in the measure? What, then, is the measure? (Ans. The measure is a *ten* of 1-inch cubes.) Observe closely the number of inch cubes I place *beside the measure* and then build *into the solid*. As I proceed George may recite.

"In 5 1-inch cubes there is *one half* of a *ten* of inch cubes." In 10 1-inch cubes there is *one ten* of inch cubes." "In 14 1-inch cubes there are *one* and *two fifths tens* of inch cubes."

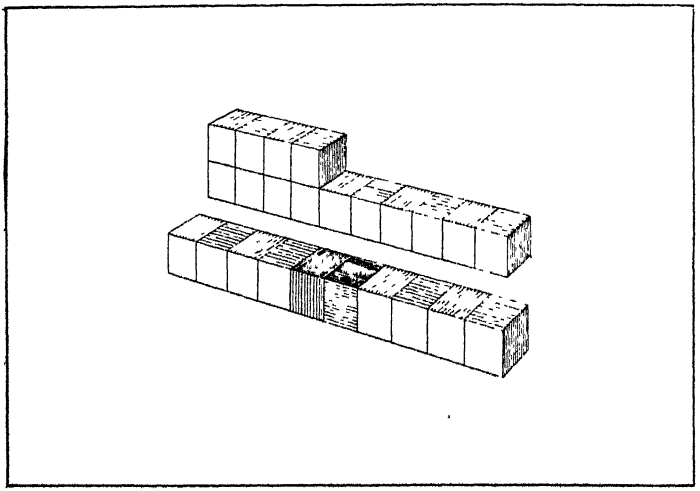


Fig. 1.

"In 20 1-inch cubes there are *two tens* of 1-inch cubes." "In 26 1-inch cubes there are *two* and *three fifths tens* of inch cubes." "In 30 1-inch cubes there are *three tens* of inch cubes."

DESK WORK.

90 pt. ÷ 1 qt. =	$\frac{1}{2}$ of 90 pt. = pt.
45 qt. − 2 qt. = pt.	$\frac{1}{3}$ of 90 pt. = qt.
44 qt. + pt. = 90 pt.	$\frac{1}{4}$ of 90 pt. = qt.
8 qt. × 5 = pt.	$\frac{1}{5}$ of 90 pt. = pt.
9 qt. × 5 = pt.	$\frac{1}{9}$ of 90 pt. = qt.
21 qt. × 4 = pt.	$\frac{1}{10}$ of 90 pt. = pt.
45 qt. − 84 pt. = pt.	

Jennie, recite.

"In 38 1-inch cubes there are *three* and *four fifths tens* of inch cubes." "In 40 1-inch cubes there are *four tens* of inch cubes." "In 50 1-inch cubes there are *five tens* of inch cubes." "In 55 1-inch cubes there are *five* and *one half tens* of inch cubes."

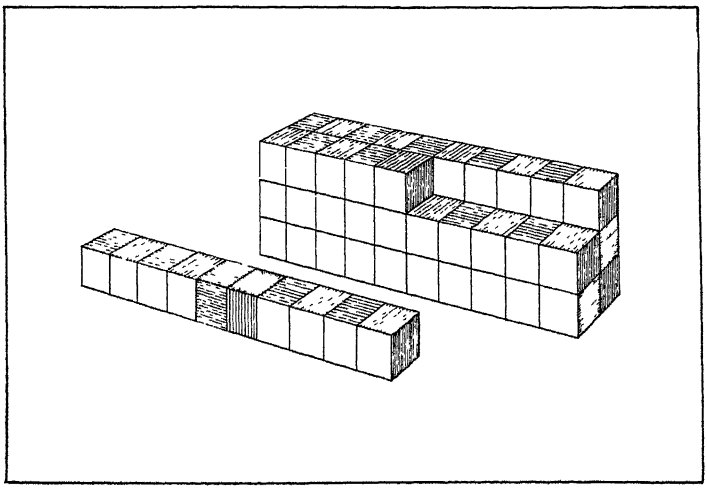


Fig. 2.

DESK WORK.

90 ft. ÷ 1 yd. =	$\frac{1}{2}$ of 90 ft. = yd.
30 yd. − 3 ft. = ft.	$\frac{1}{3}$ of 90 ft. = yd.
90 ft. − 2 yd. = ft.	$\frac{1}{5}$ of 90 ft. = yd.
27 yd. × 3 = ft.	$\frac{1}{6}$ of 90 ft. = yd.
81 ft. + yd. = 90 ft.	$\frac{1}{9}$ of 90 ft. = yd.
18 ft. × 5 = yd.	$\frac{1}{10}$ of 90 ft. = yd.
15 ft. × 6 = yd.	$\frac{7}{10}$ of 90 ft. = yd.

"In 60 1-inch cubes there are *six tens* of 1-inch cubes." "In 70 1-inch cubes there are *seven tens* of inch cubes." "In 80 1-inch cubes there are *eight tens* of inch cubes." "In 82 1-inch cubes there are *eight* and *one fifth tens* of inch cubes."

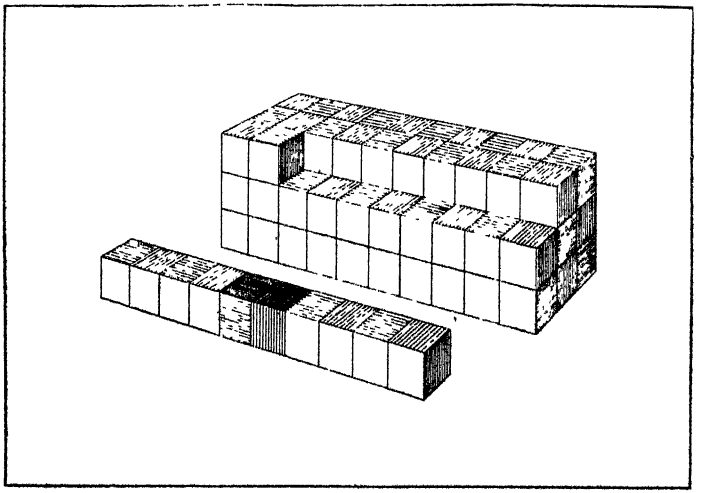


Fig. 3.

DESK WORK.

90 ct. ÷ 1 nickel =	$\frac{1}{2}$ of 90 ct. = ct.
90 ct. − 1 nic. = ct.	$\frac{1}{3}$ of 90 ct. = di.
90 ct. ÷ 1 dime =	$\frac{1}{5}$ of 90 ct. = di.
90 ct. − 2 nic. = di.	$\frac{1}{6}$ of 90 ct. = ct.
90 ct. + di. = 90 ct.	$\frac{1}{9}$ of 90 ct. = di.
3 nic. × 6 = ct.	$\frac{1}{10}$ of 90 ct. = nic.
75 ct. + nic. = 9 di.	$\frac{3}{10}$ of 90 ct. = ct.
50 ct. + 40 ct. = di.	$\frac{7}{10}$ of 90 ct. = ct.

"In 90 1-inch cubes there are *nine tens* of 1-inch cubes." Then, nine times *ten* inch cubes are how many inch cubes? In 90 1-inch cubes there are how many inch cubes?

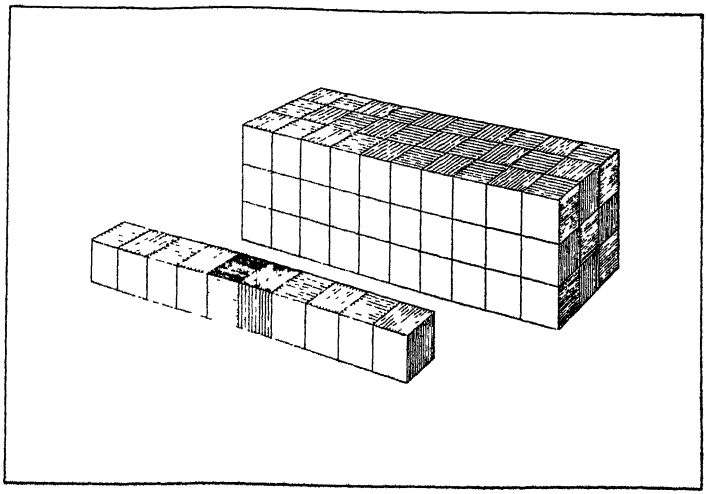


Fig. 4.

DESK WORK.

90 ft. ÷ 1-fathom =	$\frac{1}{2}$ of 90 ft. = f.
90 ft. − 1 f. = ft.	$\frac{1}{3}$ of 90 ft. = f.
90 ft. − 2 f. = f.	$\frac{1}{5}$ of 90 ft. = f.
72 ft. + f. = 90 ft.	$\frac{1}{6}$ of 90 ft. = f.
3 f. × 5 = yd.	$\frac{1}{9}$ of 90 ft. = f.
18 ft. × 5 = f.	$\frac{1}{10}$ of 90 ft. = f.
15 ft. × 6 = f.	$\frac{2}{10}$ of 90 ft. = f.

In this end section (moving out the section) there are how many 1-inch cubes? How many such sections in the solid? Then, in 90 1-inch cubes there are how many *nines* of inch cubes? *Ten* times *nine* 1-inch cubes are how many inch cubes?

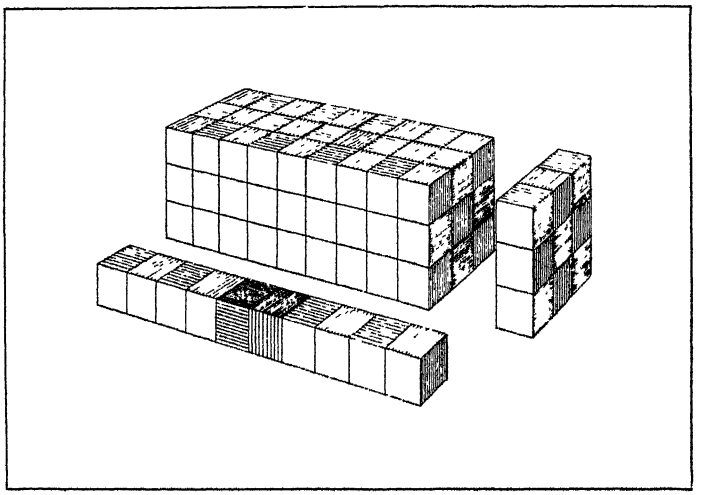


Fig. 5.

In 90 cents there are how many dimes? In 90 square feet there are how many square yards?

In the upper row of the front section there are how many 1-inch cubes? How many such rows in the section? How many 1-inch cubes in the front section? How many such sections in the solid? In 90 1-inch cubes there are how many *thirties* of inch cubes? Three times thirty 1-inch cubes are how many 1-inch cubes? What is one third of 90 inch cubes? What are two thirds of 90 inch cubes?

How many 1-inch cubes in a vertical row of the front section? How many such rows in the section? In 30 inch

cubes there are how many *threes* of inch cubes? In 60 inch cubes? In 90 inch cubes? Thirty times three inch cubes are how many inch cubes? What is one thirtieth of 90 inch cubes? In 90 feet there are how many yards? In 90 days there are how many months?

NOTE.—Divide the solid into two equal parts across its length and place one part upon the other.

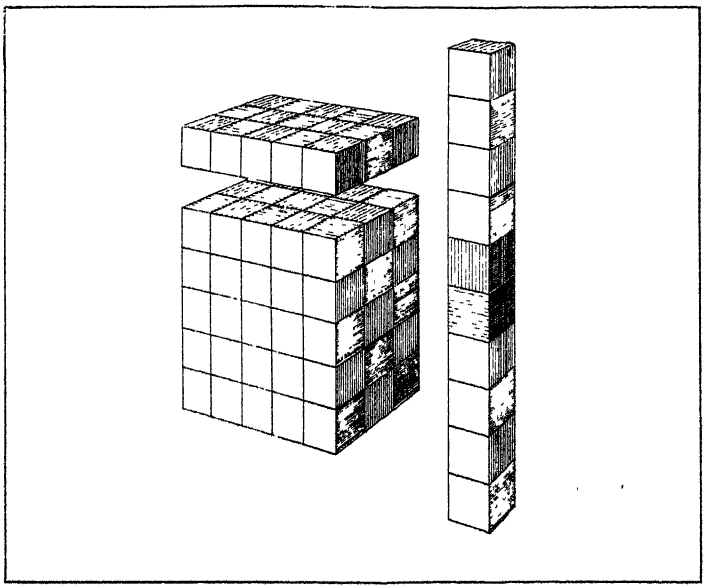


Fig. 6.

In the front row of the top section there are how many 1-inch cubes? How many such rows in the top section? How many 1-inch cubes in the top section? How many such sections in the solid? Then, in 90 1-inch cubes there are how many *fifteens* of inch cubes? How many *fives* of inch cubes in the top section? How many such sections in the solid? In 90 1-inch cubes there are how many *fives* of inch cubes? In 90 cents there are how many nickels?

In the front vertical row of the end section there are how many 1-inch cubes? In the end section there are how many such rows? How many 1-inch cubes in the end section? How many such sections in the solid? Then, in 90 1-inch cubes there are how many *eighteens* of inch cubes? Five times eighteen inch cubes are how many inch cubes?

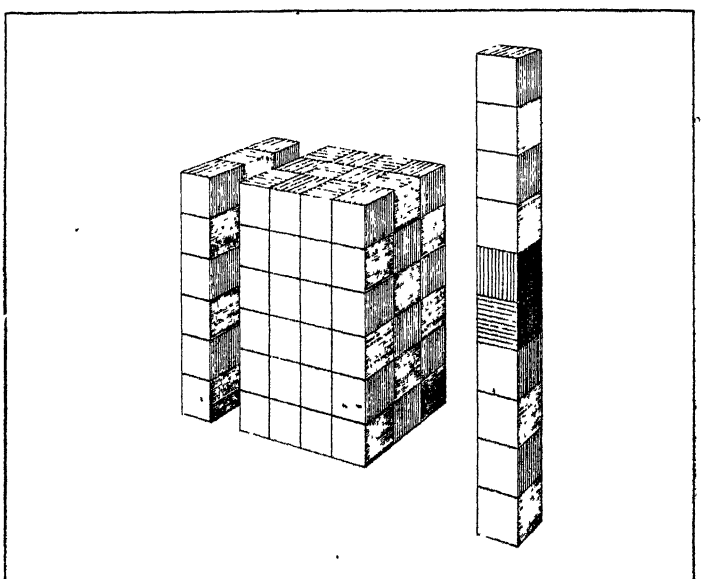


Fig. 7.

What are the factors of 90?

Answer:

$$\left.\begin{array}{r} 2 \times 45 \\ 45 \times 2 \\ 3 \times 30 \\ 30 \times 3 \\ 5 \times 18 \\ 18 \times 5 \\ 6 \times 15 \\ 15 \times 6 \\ 9 \times 10 \\ 10 \times 9 \end{array}\right\} = 90$$

Desk Work.

90 sq. ft. ÷ 1 sq. yd. =

9 sq. ft. × 9 = sq. yd.

90 sq. ft. − 81 sq. ft. = sq. yd.

9 sq. ft. × 8 = sq. ft.

90 sq. ft. − 72 sq. ft. = sq. yd.

$\frac{1}{6}$ of 90 sq. ft. = sq. yd.

$\frac{1}{10}$ of 90 sq. ft. = sq. yd.

Desk Work.

1. 45 ×2	7. 22 ×4	13. 90 −45	19. 4)90	
2. 43 ×2	8. 21 ×4	14. 90 −43	20. 5)90	
3. 41 ×2	9. 44 44	15. 90 −44	21. 6)90	
4. 30 ×3	10. 43 43	16. 90 −40	22. 9)90	
5. 29 ×3	11. 45 45	17. 2)90	23. 10)90	
6. 27 ×3	12. 45 37	18. 3)90	24. 3)87	

DRILLS FOR BLACKBOARD OR CHART.

1. 90 feet, 84 feet, 87 feet, 81 feet, 75 feet, 79 feet, 72 feet, 78 feet, 69 feet, 66 feet, 63 feet, 60 feet = yd.

2. 84 inches, 90 inches, 72 inches, 80 inches, 78 inches, 60 inches, 70 inches, 66 inches, 48 inches, 50 inches, 54 inches, 84 inches = ft.

3. 2 yards, $2\frac{1}{2}$ yards, $2\frac{1}{4}$ yards, $2\frac{1}{3}$ yards, $2\frac{1}{6}$ yards, $2\frac{5}{6}$ yards = in.

4. 20 yards, 30 yards, 25 yards, 24 yards, 27 yards, 26 yards = ft.

5. 1 rod, $\frac{1}{2}$ rod, 2 rods, 3 rods, 4 rods = ft.

6. 1 rod, 4 rods, 8 rods, 10 rods, 12 rods, 16 rods = yd.

7. 25 quarts, 30 quarts, 40 quarts, 35 quarts, 45 quarts, 22 quarts, 24 quarts, 32 quarts, 42 quarts, 36 quarts, 44 quarts, 41 quarts = pt.

8. 12 gallons, 15 gallons, 20 gallons, 16 gallons, 18 gallons, 17 gallons, 19 gallons, 25 gallons, 24 gallons, 22 gallons, 23 gallons, 21 gallons = qt.

9. 1 bushel, 2 bushels, 3 bushels = qt.

10. 1 month, 3 months, 2 months = da.

11. 7 gallons, 6 gallons, 8 gallons, $8\frac{1}{2}$ gallons, 10 gallons, 9 gallons, 12 gallons, 11 gallons = pt.

12. 88 quarts, 96 quarts, 72 quarts, 80 quarts, 64 quarts, 68 quarts, 48 quarts, 56 quarts = pk.

13. 7 weeks, 10 weeks, 8 weeks, 11 weeks, 9 weeks, 6 weeks, 12 weeks = da.

14. 6 fm., 9 fm., 8 fm., 7 fm., 10 fm., 12 fm., 11 fm. = ft.

15. 10 sq. yd., 11 sq. yd., 8 sq. yd., 7 sq. yd., 9 sq. yd., 6 sq. yd. = sq. ft.

16. 10 dimes, 8 dimes, 9 dimes, 7 dimes, 6 dimes, $7\frac{1}{2}$ dimes = ct.

17. 5 pounds, 6 pounds, 4 pounds, 3 pounds, 2 pounds = oz.

18. 1 ream, 3 reams, 2 reams, 5 reams, 4 reams = quires

19. 1 day, 2 days, 3 days, 4 days = hr.

20. 4 f'tn't, 5 f'tn't, 6 f'tn't, 7 f'tn't = da.

21. 10 nickels, 15 nickels, 12 nickels, 14 nickels, 13 nickels, 16 nickels, 20 nickels, 18 nickels, 17 nickels, 19 nickels = ct.

22. 1 hour, $1\frac{1}{2}$ hours, $1\frac{1}{4}$ hours, $1\frac{3}{4}$ hours, $1\frac{1}{3}$ hours, $1\frac{2}{3}$ hours, $1\frac{1}{10}$ hours, $1\frac{3}{10}$ hours, $1\frac{1}{6}$ hours, $1\frac{5}{6}$ hours = min.

THE YARD AS A UNIT OF REFERENCE.

Direction Write the problem, $\frac{1}{4}+\frac{1}{9}=$ what? on the board Explanation: One fourth of a yard is nine inches, one ninth of a yard is four inches; one inch is one thirty-sixth of a yard; nine inches are nine thirty-sixths of a yard, and four inches are *four* thirty-sixths of a yard; *nine* thirty-sixths of a yard and *four* thirty-sixths of a yard are *thirteen* thirty-sixths of a yard. Therefore $\frac{1}{4}+\frac{1}{9}=\frac{13}{36}$

$\frac{3}{4}+\frac{8}{9}=$	$\frac{1}{36}+\frac{1}{3}=$	$\frac{1}{18}+\frac{1}{12}=$
$\frac{8}{9}-\frac{3}{4}=$	$\frac{1}{3}-\frac{1}{36}=$	$\frac{1}{12}-\frac{1}{18}=$
$\frac{8}{9}\div\frac{3}{4}=$	$\frac{1}{3}\div\frac{1}{36}=$	$\frac{5}{12}+\frac{7}{18}=$
$\frac{3}{4}\div\frac{8}{9}=$	$\frac{1}{36}\times 3=$	$\frac{5}{12}-\frac{7}{18}=$
$\frac{8}{9}\times 1\frac{1}{2}=$	$\frac{1}{36}\div\frac{1}{3}=$	$\frac{7}{12}+\frac{17}{18}=$
$\frac{3}{4}\times\frac{1}{3}=$	$\frac{1}{36}\times 4=$	$\frac{13}{18}+\frac{5}{6}=$

LESSON 19—CLASS WORK.

THE PARTS OF (60) AN HOUR

How many minutes in an hour?
How many in a half hour?
How many in a quarter hour?
How many in three quarters of an hour?
$\frac{1}{3}$ of an hour is how many minutes?
$\frac{2}{3}$ of an hour are how many minutes?
$\frac{1}{5}$ of an hour is how many minutes?
$\frac{2}{5}$ of an hour are how many minutes?
$\frac{3}{5}$ of an hour are how many minutes?
$\frac{4}{5}$ of an hour are how many minutes?
$\frac{1}{6}$ of an hour is how many minutes?
$\frac{5}{6}$ of an hour are how many minutes?
$\frac{1}{10}$ of an hour is how many minutes?
$\frac{6}{10}$ of an hour are how many minutes?
$\frac{7}{10}$ of an hour are how many minutes?
$\frac{9}{10}$ of an hour are how many minutes?
$\frac{8}{10}$ of an hour are how many minutes?
$\frac{1}{12}$ of an hour is how many minutes?
$\frac{7}{12}$ of an hour are how many minutes?
$\frac{8}{12}$ of an hour are how many minutes?

$\frac{9}{12}$ of an hour are how many minutes?
$\frac{11}{12}$ of an hour are how many minutes?
$\frac{1}{15}$ of an hour is how many minutes?
$\frac{8}{15}$ of an hour are how many minutes?
$\frac{9}{15}$ of an hour are how many minutes?
$\frac{11}{15}$ of an hour are how many minutes?
$\frac{10}{15}$ of an hour are how many minutes?
$\frac{13}{15}$ of an hour are how many minutes?
$\frac{1}{20}$ of an hour is how many minutes?
$\frac{15}{20}$ of an hour are how many minutes?
$\frac{16}{20}$ of an hour are how many minutes?
$\frac{17}{20}$ of an hour are how many minutes?
$\frac{19}{20}$ of an hour are how many minutes?
$\frac{1}{30}$ of an hour is how many minutes?
$\frac{21}{30}$ of an hour are how many minutes?
$\frac{29}{30}$ of an hour are how many minutes?
$\frac{25}{30}$ of an hour are how many minutes?
$\frac{23}{30}$ of an hour are how many minutes?
$\frac{27}{30}$ of an hour are how many minutes?
$\frac{26}{30}$ of an hour are how many minutes?
$\frac{28}{30}$ of an hour are how many minutes?

LESSON 20—CLASS WORK.

THE HOUR AS A UNIT OF REFERENCE.

NOTE.—Write on the board the problem, $\frac{1}{4} + \frac{1}{15} =$ what? Let the following explanation be worked out: One fourth of an hour is fifteen minutes; one fifteenth of an hour is four minutes; one minute is one sixtieth of an hour; fifteen minutes are fifteen sixtieths of an hour, and four minutes are four sixtieths of an hour; fifteen sixtieths of an hour and four sixtieths of an hour are nineteen sixtieths of an hour. Therefore,

$\frac{1}{2} - \frac{10}{60} =$ $\quad$ $\frac{3}{20} + \frac{1}{6} =$ $\quad$ $\frac{1}{4} + \frac{1}{30} =$
$\frac{1}{2} + \frac{10}{60} =$ $\quad$ $\frac{1}{6} - \frac{3}{20} =$ $\quad$ $\frac{1}{4} - \frac{1}{12} =$
$\frac{1}{20} + \frac{1}{3} =$ $\quad$ $\frac{1}{6} \div \frac{3}{20} =$ $\quad$ $\frac{1}{12} - \frac{1}{15} =$
$\frac{7}{20} - \frac{1}{3} =$ $\quad$ $\frac{3}{20} \div \frac{1}{6} =$ $\quad$ $\frac{1}{20} + \frac{1}{30} =$
$\frac{7}{20} + \frac{1}{3} =$ $\quad$ $\frac{3}{20} \times 5 =$ $\quad$ $\frac{1}{30} \div \frac{1}{60} =$
$\frac{1}{3} \div \frac{7}{20} =$ $\quad$ $\frac{3}{20} \times \frac{1}{3} =$ $\quad$ $\frac{1}{20} \div \frac{1}{60} =$

MULTIPLICATION TABLE.

(50 TO 100.)

6 times 9
9 times 6 } = 54

5 times 11
11 times 5 } = 55

7 times 8
8 times 7 } = 56

6 times 10
10 times 6
5 times 12
12 times 5 } = 60

7 times 9
9 times 7 } = 63

8 times 8 = 64

6 times 11
11 times 6 } = 66

7 times 10
10 times 7 } = 70

8 times 9
9 times 8
6 times 12
12 times 6 } = 72

7 times 11
11 times 7 } = 77

8 times 10
10 times 8 } = 80

9 times 9 = 81

7 times 12
12 times 7 } = 84

8 times 11
11 times 8 } = 88

9 times 10
10 times 9 } = 90

8 times 12
12 times 8 } = 96

9 times 11
11 times 9 } = 99

10 times 10 = 100

A MEMORY LESSON.

9 times 12
12 times 9 } = 108

10 times 11
11 times 10 } = 110

10 times 12
12 times 10 } = 120

11 times 11 = 121

11 times 12
12 times 11 } = 132

12 times 12 = 144

CHAPTER XX.

Notation and Numeration—Addition—Percentage.

Lesson 21—Class Work.

NOTATION AND NUMERATION.

Note.—Teach thoroughly and fully each point in the lessons of Chapters XX., XXI., and XXII.

1. A *unit* is a single thing of any kind. A *foot*, a *gallon*, a bushel, a pound are units.

2. A *number* is an expression denoting one or more units; as, 3 (feet), 4 (gallons), 5 (bushels), 6 (pounds).

3. Numbers are formed of the characters 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. These characters are called figures. The last one is called zero, cipher, or naught. The others are sometimes called digits. This method of expressing numbers is known as the Arabic method.

4. Numbers are also formed of letters. The letters used to express numbers are: I, V, X, L, C, D, M, and their values are respectively 1, 5, 10, 50, 100, 500, 1000. This is the Roman method of writing numbers.

5. Table of Roman Notation:

I..... 1	XIV.....14	LX..... 60
II..... 2	XV.....15	LXX..... 70
III..... 3	XVI.....16	LXXX..... 80
IV..... 4	XVII.....17	XC..... 90
V..... 5	XVIII.....18	C..... 100
VI..... 6	XIX.....19	CC..... 200

VII... 7	XX 20	CCXL .. . 240
VIII 8	XXI 21	CCC . 300
IX 9	XXII.. 22	D 500
X 10	XXV 25	DC 600
XI 11	XXX 30	M ... .1000
XII.. 12	XL .. .. .40	MM ... .2000
XIII. 13	L 50	MDCCCXCIX..... 1899

LESSON 22—CLASS WORK.

6. Write the following numbers, using the Roman method:

35, 49, 53, 56, 64, 78, 71, 82, 97, 99, 100.

7. Writing numbers in figures or letters is called *notation*.

8. Reading numbers is called *numeration*.

9. Count by 100's from 0 to 1000.
Count by 50's from 0 to 1000.
Count by 25's from 0 to 1000.
Count by 20's from 0 to 1000.
Count by 10's from 0 to 1000.
Count by 5's from 0 to 200.
Count by 5's from 200 to 400.
Count by 5's from 400 to 600.
Count by 5's from 600 to 800.
Count by 5's from 800 to 1000.

10. How many 1's in 10? How many 10's in 100? How many 100's in 1000?

The *position* of a figure is called an *order*. The first *order* is the one to the right and is known as the *units'* order. The second order is the *tens'* order; the third, the *hundreds'* order, and the fourth, the *thousands'* order.

Zero is used to fill vacant orders.

Write *1* unit. Write *1 ten* of units. Write *1 hundred* of units. Write *1 thousand* of units. Write *2* units. Write *2 tens* of units. Write *2 hundreds* of units. Write *2 thousands* of units. Write *3* units. Write *3 tens* of units. Write *3 hundreds* of units. Write *3 thousands* of units.

Write:—

4 units; 4 tens; 4 hundreds; 4 thousands.
5 units; 5 tens; 5 hundreds; 5 thousands.
6 units; 6 tens; 6 hundreds; 6 thousands.
7 units; 7 tens; 7 hundreds; 7 thousands.
8 units; 8 tens; 8 hundreds; 8 thousands.
9 units; 9 tens; 9 hundreds; 9 thousands.

How many *orders* does it take to write 6 units? Are there any vacant orders? How many orders does it take to write *6 tens*? Are there any vacant orders? How many? What is the vacant order? How many orders does it take to write 6 hundreds? Are there any vacant orders? What are they? How many orders does it take to write 6 thousands? How many vacant orders are there? What are they? Write 6 in each of the first four orders beginning with the fourth order. Read the number enumerating the orders. (Ans. 6 thousands, 6 hundreds, 6 tens, 6 units.) Read the number in a similar way. (Ans. Six thousand, six hundred, sixty-six.)

Write four in the fourth order, 3 in the third order, 2 in the second order, and 1 in the first order. Read the number, enumerating the orders. Read without enumerating the orders.

Write 4 in the fourth, 3 in the third, 2 in the second, and zero in the first. Read, enumerating the orders. Read, not enumerating orders.

Write 4 in the fourth, 3 in the third, 0 in the second, 0 in the first. Read, enumerating the orders. (Ans. 4 thousands, 3 hundreds, no tens, no units.) Read without naming the orders.

Write 4 in the fourth and 0 in each of the other orders. Read in both ways.

Write and read in both ways:

9 in the 4th, 8 in the 3d, 7 in the 2d, 6 in the 1st.
9 in the 4th, 8 in the 3d, 7 in the 2d, 0 in the 1st.
9 in the 4th, 8 in the 3d, 0 in the 2d, 0 in the 1st.
9 in the 4th, 0 in the 3d, 0 in the 2d, 0 in the 1st.
8 in the 4th, 0 in the 3d, 0 in the 2d, 8 in the 1st.
8 in the 4th, 0 in the 3d, 8 in the 2d, 8 in the 1st.
8 in the 4th, 8 in the 3d, 8 in the 2d, 8 in the 1st.
7 in the 4th, 0 in the 3d, 7 in the 2d, 0 in the 1st.
7 in the 4th, 7 in the 3d, 0 in the 2d, 0 in the 1st.
7 in the 4th, 0 in the 3d, 0 in the 2d, 7 in the 1st.

Lesson 23—Class Work.

11. Write:—

8391	5638	6789
7741	5008	6009
9352	5038	6089
9052	3600	6709
9002	4237	5307
9300	5725	5805

12. Read in two ways (from the blackboard) the following:—

456	1333	4567
804	2	7654
840	20	7004
70	200	7600
707	2000	7060
777	2222	7604

Lesson 24—Class Work.

13. Write in figures (from the blackboard):—
Six thousand five hundred nine.
Nine thousand nine hundred eleven.
Seven thousand four hundred eighty-nine.
Eighty thousand four hundred forty-four.
Five thousand four hundred eighty-six.
Six thousand six.
Eight thousand sixty.
Seven thousand six hundred six.
Four thousand five hundred thirty-two.
Four thousand two.
Four thousand twenty-two.
Four thousand two hundred two.
Four thousand two hundred twenty-two.
Three thousand one hundred ten.
Three thousand one.
Three thousand one hundred.
Three thousand one hundred eleven.
Six thousand four hundred fifty-three.
Two thousand two hundred twenty-two.
One thousand two hundred thirty-four.
One thousand four.
One thousand forty.

Lesson 25—Class Work.

14. For convenience in writing and reading numbers they are separated by commas into groups of three orders each. These groups are called periods. The *first* period consists of the first three orders and is called the *units' period.* The *second* period consists of the next three orders and is called the *thousands'* period. The *third* period is the *millions'* period. The fourth period is the *billions'* period. The orders of the second period are thousands, ten-thousands, hundred-thousands. Those of the third

are millions, ten-millions, hundred-millions. Those of the fourth are billions, ten-billions, hundred-billions.

NOTE —The teacher will impress the above by *illustrations* and *drill* at this point.

LESSON 26—CLASS WORK.

15. NOTE —These numbers are to be placed on the board that the pupils may (1) separate them into *periods*, (2) name the periods, (3) name the orders; (4) read the numbers.

999888777	990880770	900800700
9000000	98000000	987000000
12135673	3045216825	5216839542
3245437618	77065214897	252621356785
562534435890	440440440440	222333444555
600600600600	333222111444	660770880990
505505505505	666555444333	77025064032

LESSON 27—CLASS WORK.

16. NOTE —The following are to be *written in figures*, *pointed*, and *read:*

One hundred sixty-two thousand.

One hundred twenty thousand two hundred sixty-one.

Four hundred seventy-four thousand three hundred thirty-five.

Eight hundred fifty-three thousand seven hundred fifty.

Two million one hundred thirteen.

Four million four thousand forty.

Five billion five million five thousand five.

Nine hundred seven million eight hundred five thousand seventy-five.

Fifty-seven billion forty-four million ninety-three thousand eighty-four.

Five hundred six billion two million five thousand one.

Ninety billion eighty million seventy thousand sixty.

NOTE.—The work in notation and numeration is to be *extended* and *continued* throughout the year.

LESSON 28—CLASS WORK.

ADDITION.

17. The result obtained by combining numbers or putting them together, is called the *sum*. If 4 and 3 are combined into one quantity their sum is 7. *Addition* is finding the sum of two or more numbers.

18. The *sign* of addition is +, plus. It indicates that the numbers between which it is placed are to be added.

19. Only numbers of the same *kind* can be added. The sum of 5 pounds + 5 pounds + 3 pounds equals 13 pounds.

20. For the word, equals, the sign, =, is used. It is often called the sign of equality. It shows that the quantities between which it stands have the same value.

21. Add by 3's from 0 to 99.
Add by 3's from 2 to 101.
Add by 3's from 4 to 103.
Add by 3's from 5 to 104.
Add by 3's from 7 to 106.
Add by 3's from 8 to 107.
Add by 4's from 0 to 100.
Add by 4's from 1 to 101.
Add by 4's from 2 to 102.
Add by 4's from 3 to 103.
Add by 4's from 5 to 105.
Add by 4's from 7 to 107.
Add by 5's from 0 to 100.
Add by 5's from 1 to 101.
Add by 5's from 2 to 102.
Add by 5's from 3 to 103.
Add by 5's from 4 to 104.
Add by 6's from 0 to 102.

Add by 6's from 1 to 103.
Add by 6's from 2 to 104.
Add by 6's from 3 to 105.
Add by 6's from 4 to 106.
Add by 6's from 5 to 107.
Add by 7's from 0 to 98.
Add by 7's from 2 to 100.
Add by 7's from 1 to 99.
Add by 7's from 3 to 101.
Add by 7's from 5 to 103.
Add by 8's from 0 to 96.
Add by 8's from 1 to 97.
Add by 8's from 3 to 99.
Add by 9's from 0 to 99.
Add by 9's from 1 to 100.
Add by 9's from 3 to 102.
Add by 9's from 7 to 106.
Add by 10's from 0 to 100.
Add by 10's from 1 to 101.
Add by 10's from 2 to 102.
Add by 10's from 3 to 103.
Add by 10's from 4 to 104.
Add by 10's from 5 to 105.
Add by 10's from 6 to 106.

22. Exercises in column addition:—

1.	2.	3.	4.	5.	6.	7.	8.
1	9	8	7	6	5	4	3
2	1	1	1	1	1	1	8
3	2	2	2	2	2	2	7
4	3	3	3	3	3	3	9
5	4	4	4	4	4	7	6
6	5	5	5	5	6	6	4
7	6	6	6	7	9	9	2
8	7	7	8	8	7	8	1
9	8	9	9	9	8	5	5

23. Exercises in column addition:—

1.	2.	3.	4.	5.	6.	7.	8.
6	9	6	8	7	5	4	3
9	7	8	9	8	8	9	6
8	5	8	4	4	9	7	9
7	8	6	7	9	7	8	8
5	6	7	6	7	6	6	4
9	9	9	8	6	8	4	2
7	4	4	7	5	9	9	6
8	7	8	5	9	5	5	7
4	6	7	9	4	6	7	8
9	8	9	6	8	7	8	9

NOTE.—Continue these drills until pupils have acquired accuracy and facility.

LESSON 29—CLASS WORK.

24. Exercises in addition:—

1. 34
   65
   23
   48
   25
   195

Explanation: 5, 13, 16, 21, 25; the sum of the units is 25—5 units and 2 tens; write the 5 in units' order and add the 2 in tens' order; 2, 4, 8, 10, 16, 19; the sum of the tens is 19—9 tens and 1 hundred; write the 9 in tens' order and the 1 in hundreds' order. Ans. 195.

2.	3.	4.	5.	6.	7.
25	89	73	78	59	49
63	43	29	89	68	85
48	72	36	97	77	78
24	89	75	68	96	26
37	64	82	86	85	92

DESK WORK.

25. Add:—

1.	2.	3.	4.	5.
834	34	846	48	787
527	290	567	196	998
635	107	972	978	879
258	988	785	45	947
192	65	999	474	789

DESK WORK.

26. Add:—

6.	7.	8.	9.
1250	8469	9784	4798
125	9572	6795	5267
78	788	7649	6529
9987	3697	5828	9144
6858	7876	8175	4566
7092	6789	6549	7987

DESK WORK.

27. Add:—

10.	11.	12.
24862	1286549	65479821
354987	80325	72658973
6013	976898	4653857
726879	5453679	69786
1576384	9592897	289348675

LESSON 30—CLASS WORK.

PERCENTAGE.

28. Children, you have learned that in a bushel there are two halves of a bushel; that a gallon has four fourths of a gallon in it; that a foot is three thirds of itself. Now, I want to tell you something you have not heard before. It is this: A bushel is one hundred per cent of itself. A gallon has one hundred per cent of a gallon in it. It takes one hundred per cent of a foot to make a foot.

What per cent of a yard makes a yard? A pound is what per cent of itself? A ream is what per cent of itself? What per cent of a peck makes a peck? What per cent of a day makes a day? A gallon is what per cent of a gallon?

If it takes a hundred per cent of a bushel to make a bushel, a half bushel is what per cent of a bushel? A half gallon is what per cent of a gallon? Six inches are what per cent of a foot? Eight ounces are what per cent of a pound? Twelve hours are what per cent of a day? Ten quires are what per cent of a ream? What per cent of a gallon is one quart? What per cent is a pint of a quart? A peck is what per cent of a bushel? Two pecks are what

per cent of a bushel? Three pecks? Four pecks? Three quarts are what per cent of a gallon? Four ounces are what per cent of a pound? Twelve ounces? What per cent of a foot are nine inches? A foot is what per cent of a yard? Two feet? Three feet? Six hours are what per cent of a day? Eighteen hours are what per cent of a day? Five quires are what per cent of a ream? Fifteen quires? Thirty minutes are what per cent of an hour? Fifteen minutes? Twenty minutes are what per cent of an hour? Forty minutes? Ten days are what per cent of a month? Twenty days? Seven days are what per cent of a fortnight?

LESSON 31—CLASS WORK.

29. What is 50 per cent of a bushel of wheat? What is 25 per cent of a bushel of wheat? What is 50 per cent of a half-bushel of wheat? What is 50 per cent of a gallon of milk? What is 25 per cent of a gallon of milk? A pint is what *part* of a quart? A pint is what per cent of a quart? A pint is what *part* of a gallon? A pint is what per cent of a gallon? What is $33\frac{1}{3}$ per cent of a foot? 50 per cent of a foot? 25 per cent of a foot? 75 per cent of a foot? $66\frac{2}{3}$ per cent of a foot? $12\frac{1}{2}$ per cent of a foot? What is $33\frac{1}{3}$ per cent of a yard? $66\frac{2}{3}$ per cent of a yard? What is 50 per cent of a yard?

What is $12\frac{1}{2}$ per cent of a peck of corn?
What is 25 per cent of a peck of corn?
What is 50 per cent of a peck of corn?
What is 75 per cent of a peck of corn?
What is 100 per cent of a peck of corn?
What is 100 per cent of a pound of butter?
What is 50 per cent of a pound of butter?
What is 25 per cent of a pound of butter?
What is 75 per cent of a pound of butter?
What is $12\frac{1}{2}$ per cent of a pound of butter?
What is $37\frac{1}{2}$ per cent of a pound of butter?

CHAPTER XXI.

Subtraction—Multiplication—Percentage.

Lesson 32—Class Work.

SUBTRACTION.

1. Nine is greater than seven; eighteen is greater than thirteen. If seven be taken from nine the result, two, will show that nine is greater than seven by two. If thirteen be taken from eighteen the result, five, will show that eighteen is greater than thirteen by five.

2. The result obtained by taking one number from another is called the *difference*.

3. Taking one number from another is *subtraction*.

4. The number *subtracted* is the *subtrahend*.

5. The number *subtracted from* is the *minuend*.

6. The *sign* of subtraction is −, minus. It means that the quantity on its right is to be taken from the one on its left.

7. The difference can be found between *like* numbers only.

8. Subtract 354 from 968.

968
354
614

Explanation: 4 units from 8 units leave 4 units, write the 4 units in units' order; 5 tens from 6 tens leave 1 ten, write the 1 ten in tens' order; 3 hundreds from 9 hundreds leave 6 hundreds, write the 6 hundreds in the hundreds' order. Ans. 614. (The minuend is 968, the subtrahend is 354, and the difference 614.)

9. Exercises in subtraction:—

1. 564	2. 759	3. 869	4. 468
323	423	537	222
5. 897	6. 658	7. 8694	8. 9768
853	505	3461	8533

Lesson 33—Class Work.

10. From 51 subtract 28:—

51 Explanation: 8 units cannot be taken from 1 unit;
28 taking 1 ten from 5 tens and adding it to the 1 unit
23 make 11 units; 8 units from 11 units leave 3 units; write the 3 units in units' order; 2 tens from 4 tens leave 2 tens; write the 2 tens in tens' order. Ans. 23. (The minuend, or the number subtracted from, is 51; the subtrahend, or the number subtracted, is 28; the difference or remainder, is 23.)

11. Exercises in subtraction:—

1. 41	2. 65	3. 53	4. 62	5. 34
19	26	25	44	17
6. 77	7. 86	8. 81	9. 92	10. 95
58	28	22	57	46

Lesson 34—Class Work.

12. From 1725 subtract 352:—

1725 Explanation: 2 units from 5 units leave 3 units;
352 write the 3 units in units' order; 5 tens cannot be taken from 2 tens; taking 1 hundred, or 10 tens from 7 hundreds and adding it to 2 tens make 12 tens; 5 tens from 12 tens leave 7 tens; write the 7 tens in tens' order; 3 hundreds from 6 hundreds leave 3 hundreds; write

the 3 hundreds in hundreds' order; no thousands from 1 thousand leaves 1 thousand; write the 1 thousand in thousands' order. Ans. 1373. (The subtrahend is 352; the minuend is 1725, and the difference, or remainder, 1373.)

13. Exercises in subtraction:—

1.	2.	3.	4.
4270	3525	2111	1265
1352	1636	1222	456

DESK WORK.

1.	2.	3.	4.
4342	7356	6428	8560
−1453	−3327	−5439	−6783
5.	**6.**	**7.**	**8.**
5222	9000	8234	9634
−1888	−3123	−7546	−1645

LESSON 35—CLASS WORK.

NOTE.—14. In the sixth example above, the minuend is changed from 9000 to its equivalent, $899_{[10]}$. Explanation: 1 thousand from 9 thousand leaves *8* thousand, the 1 thousand = 10 hundreds; 1 hundred from the 10 hundreds leaves *9* hundreds, the 1 hundred = 10 tens; 1 ten from 10 tens leaves *9* tens, the 1 ten = 10 units. The minuend becomes $899_{[10]}$.

Change the following minuends and with the same subtrahend (3123) find the difference: 8000; 70000; 60000; 600000; 50000; 5000000; 45000; 350000; 29000; 180000.

15. Exercises in subtraction:

1. From 7040316 take 5153278.
2. From 9642716 take 653918.
3. From 7875430 take 3986065.
4. From 8420431 take 836507.
5. From 90070080 take 50460509.
6. From 6748075 take 99988.

LESSON 36—CLASS WORK.

16. To prove subtraction add the difference and subtrahend. The work is correct if the sum is equal to the minuend.

Subtract 785958 from each of the following and prove: 2456785; 21565493; 965341; 235656351; 834563; 10000000; 5003004.

LESSON 37—CLASS WORK.

MULTIPLICATION.

17. Multiplication is repeating a quantity a certain number of times. $\$5 \times 3 = \$15$. In the result, \$15, the quantity, \$5, is repeated three times.

18. The number repeated is called the *multiplicand.*

19. The number showing how many times the multiplicand is repeated is called the *multiplier.*

20. The result obtained by multiplication is the *product.*

In the example above \$5 is the multiplicand, 3 the multiplier, and \$15 the product.

21. The sign of multiplication, $\times$, is read *multiplied by* or *times.* If the multiplicand is read first the sign should be read *multiplied by*, but if the multiplier is read first the sign should be read *times.*

22. A *concrete* number is one that designates a particular kind of unit; as 3 gallons, 5 pounds, \$9.

23. An *abstract* number is one that does not designate a particular kind of unit; as, 4, 6, 13.

24. The *factors* of a *product* are the numbers which multiplied together make the product.

25. The multiplicand and product are like numbers.

26. The multiplier as such is always an abstract number.

LESSON 38—CLASS WORK.

27. Multiply 658 by 2.

658 Explanation: 2 times 8 units are 16 units—1 ten
2 and 6 units; write the 6 units in units' order and
1316 add the 1 ten; 2 times 5 tens are 10 tens, 10 tens and 1 ten are 11 tens—1 ten and 1 hundred—write the 1 ten in tens' order and add the 1 hundred; 2 times 6 hundreds are 12 hundreds, 12 hundreds and 1 hundred are 13 hundreds—3 hundreds and 1 thousand—write the 3 hundreds in hundreds' place and the 1 thousand in thousands' place. Ans. 1316. (658 is the multiplicand, 2 is the multiplier, and 1316, the product.)

28. EXERCISES.

1.	739 3	2.	975 2	3.	867 4	4.	689 5
5.	7695 6	6.	85096 7	7.	90483 8	8.	76245 9
9.	568732 6	10.	457698 8	11.	908072 7	12.	8798634 9

LESSON 39—CLASS WORK.

29. Multiply 536 by 42.

536 Explanation: 2 times 536 units are 1072 units;
42 write 1072 as the first partial product; 4 *tens* (40)
1072 times 536 are 2144 *tens* or 21440, write this as the
2144 second partial product; adding the partial products,
22512 the entire product is 22512.

30. EXERCISES.

1. 428 53	2. 567 38	3. 854 56	4. 947 83
5. 3427 69	6. 4867 85	7. 5962 78	8. 9867 96
9. 65728 77	10. 87678 58	11. 71986 79	12. 809070 95
13. 67954 348	14. 62859 497	15. 95867 568	16. 906872 697

DESK WORK.

Multiply:—

17. 81756 by 34
18. 41367 by 12
19. 487 by 24
20. 593 by 35
21. 473 by 37
22. 3675 by 48
23. 3413 by 41
24. 4681 by 73
25. 57031 by 42
26. 63485 by 75
27. 85316 by 62
28. 38313 by 45
29. 39615 by 35
30. 23723 by 63
31. 51085 by 73
32. 72892 by 234
33. 43521 by 123
34. 56427 by 506
35. 27804 by 426
36. 58206 by 469
37. 132367 by 578
38. 249164 by 392
39. 191642 by 619
40. 292573 by 529
41. 345678 by 678
42. 543876 by 792
43. 453786 by 478
44. 809075 by 298
45. 967684 by 876
46. 786594 by 679

LESSON 40—CLASS WORK.

31. To multiply by 10, 100, 1000, etc., annex as many ciphers to the right of the multiplicand as there are ciphers in the multiplier.

EXERCISES.

1. Multiply 324 by 10
2. Multiply 65 by 10
3. Multiply 6 by 10
4. Multiply 2034 by 10
5. Multiply 7840 by 100
6. Multiply 40 by 100
7. Multiply 653 by 1000
8. Multiply 72 by 1000

Lesson 41—Class Work.

32. To multiply by any number having ciphers on its right multiply *without regard to the ciphers* and annex to the product thus obtained as many ciphers as there are on the right of the multiplier.

EXERCISES.

1. Multiply 465 by 400
2. 378 by 2300
3. 1672 by 120
4. Multiply 8796 by 5000
5. 9564 by 6500
6. 8769 by 7800

Lesson 42—Class Work.

PERCENTAGE.

33. 320 rods = 1 mile.

$\frac{1}{2}$ mile = rods = % of mile.

$\frac{1}{4}$ mile = rods = % of mile.

$\frac{1}{8}$ mile = rods = % of mile.

$\frac{1}{16}$ mile = rods = % of mile.

$\frac{1}{32}$ mile = rods = % of mile.

10 % of 1 mile = rods = part of mile.

20 % of 1 mile = rods = part of mile.

30 % of 1 mile = rods = part of mile.

40 % of 1 mile = rods = part of mile.

50 % of 1 mile = rods = part of mile.

70 % of 1 mile = rods = part of mile.

60 % of 1 mile = rods = part of mile.

80 % of 1 mile = rods = part of mile.

90 % of 1 mile = rods = part of mile.

100 % of 1 mile = rods = part of mile.

LESSON 43—CLASS WORK.

NOTE.—Lead pupils to find how many square inches in a square foot.

34. 144 square inches = 1 square foot.

$\frac{1}{2}$ sq. ft. = sq. in. = % of sq. ft.
$\frac{1}{4}$ sq. ft. = sq. in. = % of sq. ft.
$\frac{1}{8}$ sq. ft. = sq. in. = % of sq. ft.
$\frac{1}{16}$ sq. ft. = sq. in. = % of sq. ft.
$\frac{1}{32}$ sq. ft. = sq. in. = % of sq. ft.
$\frac{1}{3}$ sq. ft. = sq. in. = % of sq. ft.
$\frac{1}{6}$ sq. ft. = sq. in. = % of sq. ft.
$\frac{1}{12}$ sq. ft. = sq. in. = % of sq. ft.
$\frac{1}{24}$ sq. ft. = sq. in. = % of sq. ft.

Fifty per cent of 1 square foot is how many square inches?

Twenty-five per cent of 1 square foot is how many square inches? Seventy-five per cent?

Thirty-three and one third per cent of a square foot is how many square inches? Sixty-six and two thirds per cent?

LESSON 44—CLASS WORK.

35. 640 acres = 1 square mile.

What is 10 % of a square mile in acres?
What is 20 % of a square mile in acres?
What is 50 % of a square mile in acres?
What is 25 % of a square mile in acres?
What is 75 % of a square mile in acres?
What is $12\frac{1}{2}$ % of a square mile in acres?
What is $37\frac{1}{2}$ % of a square mile in acres?
What is $62\frac{1}{2}$ % of a square mile in acres?
What is $6\frac{1}{4}$ % of a square mile in acres?
Twenty acres are what % of a square mile?

LESSON 45—CLASS WORK.

36. 160 square rods = 1 acre.

$\frac{1}{2}$ of 1 acre = sq. rd.	$\frac{3}{5}$ of 1 acre = sq. rd.
$\frac{1}{10}$ of 1 acre = sq. rd.	$\frac{4}{5}$ of 1 acre = sq. rd.
$\frac{1}{4}$ of 1 acre = sq. rd.	$\frac{1}{8}$ of 1 acre = sq. rd.
$\frac{3}{4}$ of 1 acre = sq. rd.	$\frac{3}{8}$ of 1 acre = sq. rd.
$\frac{3}{10}$ of 1 acre = sq. rd.	$\frac{1}{16}$ of 1 acre = sq. rd.
$\frac{1}{5}$ of 1 acre = sq. rd.	$\frac{1}{20}$ of 1 acre = sq. rd.
$\frac{2}{5}$ of 1 acre = sq. rd.	$\frac{1}{32}$ of 1 acre = sq. rd.
What is 10 % of 1 acre?	What is 20 % of 1 acre?
What is 40 % of 1 acre?	What is 60 % of 1 acre?
What is 30 % of 1 acre?	What is 80 % of 1 acre?
What is 100 % of 1 acre?	What is 50 % of 1 acre?
What is 70 % of 1 acre?	What is 25 % of 1 acre?
What is 75 % of 1 acre?	What is $12\frac{1}{2}$ % of 1 acre?
What is $37\frac{1}{2}$ % of 1 acre?	What is $6\frac{1}{4}$ % of 1 acre?
What is $3\frac{1}{8}$ % of 1 acre?	What is 90 % of 1 acre?

80 sq. rd. are what % of an acre? 40 sq. rd. are what % of an acre?

CHAPTER XXII.

Short Division—Long Division—Percentage.

LESSON 46—CLASS WORK.

1. How many 4-pound packages can be made from a sack of coffee containing 128 pounds?

How many 4's of pounds in 128 pounds? 128 pounds are how many times 4 pounds?

How many times is 4 pounds contained in 128 pounds?

128 pounds ÷ 4 pounds = what?

A cord is 123 feet long; into how many 3-foot lengths may it be cut? How many yards long is the cord?

How many 3's of feet in 123 feet? 123 feet are how many times 3 feet? How many times is 3 feet contained in 123 feet?

123 feet ÷ 3 feet = what?

A Sunday-school teacher, by giving each member of his class a nickel, paid out 75 cents; how many pupils did he have?

How many 5's of cents in 75 cents? 75 cents are how many times 5 cents?

How many times is 5 cents (a nickel) contained in 75 cents?

75 cents ÷ 5 cents = what?

Division is finding how many times one number is contained in another.

2. A man pays 7 laborers $84, giving them equal amounts; how much does each receive?

NOTE.—In this analysis the expression "$\frac{1}{7}$ of" should be used, not "7 is contained times in".

What is $\frac{1}{7}$ of $84? $84 ÷ 7 = what?

If there had been 2 laborers, what would each have received?

What is $\frac{1}{2}$ of \$84? \$84 ÷ 2 = what?

Had there been 3 laborers, what would each have received?

What is $\frac{1}{3}$ of \$84? \$84 ÷ 3 = what?

Had there been 4 laborers, what would each have received?

What is $\frac{1}{4}$ of \$84? \$84 ÷ 4 = what?

Had there been 6 laborers, what would each have received?

What is $\frac{1}{6}$ of \$84? \$84 ÷ 6 = what?

Division is finding one of the equal parts of a number.

3. This sign ÷ indicates division. It means that the number before it is to be divided by the one after it. It is read "divided by".

The following forms also imply division: 5)285; 65)42865(. The first is the usual form for short division and the second for long division. When the number to divide by does not exceed 12, it is called *short division*; and when it does exceed 12, it is called *long division*.

4. The number *divided* is the *dividend*.

5. The number *divided by* is the *divisor*.

6. The number obtained by division is the *quotient*.

7. The part, if any, of the dividend not used in the division is the *remainder*.

8. Divide 846 by 2.

2)846 Explanation: 2 is contained in 8 hundred 4 hundred times—write the 4 in hundreds' order; 2 is contained in 4 tens 2 tens times—write the 2 in tens' order; 2 is contained in 6 units 3 times—write the 3 in units' order. Ans. 423.

NOTE.—After the solution and explanation of each problem such questions as follow should be asked:—

What is the dividend in this example? What is the divisor? What is the quotient? Is there a remainder?

How many 2's in 846? 846 are how many times 2? How many times is 2 contained in 846? What is one half of 846?

2. 256 ÷ 2 =	7. 186 ÷ 3 =	12. 376 ÷ 4 =
3. 188 ÷ 2 =	8. 549 ÷ 3 =	13. 568 ÷ 4 =
4. 634 ÷ 2 =	9. 831 ÷ 3 =	14. 988 ÷ 4 =
5. 752 ÷ 2 =	10. 477 ÷ 3 =	15. 464 ÷ 4 =
6. 930 ÷ 2 =	11. 984 ÷ 3 =	16. 756 ÷ 4 =

9. When there is a remainder, the exact quotient may be shown by placing the divisor below a horizontal line drawn under the remainder and annexing the result to the quotient already obtained.

10. DESK WORK.

1. 5)675	4. 5)831	7. 6)956	10. 6)278	13. 7)750
2. 5)542	5. 5)701	8. 6)560	11. 6)852	14. 7)1026
3. 5)659	6. 6)960	9. 6)686	12. 7)805	15. 7)8758

DESK WORK.

11. Divide the following numbers by 8, 9, 10, 11, 12:

1. 5685	6. 806407	11. 10345813
2. 5643	7. 1230565	12. 24256712
3. 10840	8. 1230875	13. 40021648
4. 9083	9. 8562926	14. 562175492
5. 340124	10. 751524	15. 907856342

DESK WORK.

NOTE.—Use 12 as a divisor in teaching the *form* of long division.

12. Divide:—

1. 144 by 1	6 2304 by	11. 331776 by 12
2. 288 by 1	7 6912 by	12. 497664 by 12
3. 576 by 1	8. 27648 by 1	13. 663552 by 12
4. 1728 by 12	9. 55296 by 12	14. 995328 by 12
5. 1152 by 12	10. 165888 by 12	15. 997056 by 12

NOTE.—Probably the greatest difficulty to be encountered in teaching long division is that of getting pupils to see the relation or approximate relation of the partial dividend to the divisor. To

overcome this difficulty let such divisors as 25, 50, and 100 be used somewhat in the following way.—

1 A short *oral* drill before a problem is given, in which the teacher may ask such questions as How many times is 25 contained in 100? How many times and with what remainder is 25 contained in 99? in 95? in 90? in 85? in 80? in 76? in 102? in 105? in 110? in 115? in 120? How many times is 25 contained in 200? How many times and with what remainder is 25 contained in 195? in 190? in 180? in 176? in 174? in 152? in 205? in 215? in 224? How many times is 25 contained in 225? This will prepare the class for the problem and enable them to divide by 25 readily

2 After a great many problems, using 25 as a divisor, then give problems with divisors a little less than 25, and then problems with divisors a little more than 25, leading pupils to make 25 the *basis* in determining the desired relation.

3. When this is finished give the *oral* drill, using 50 as the divisor, and follow *with problems* having 50 for the divisor; then introduce divisors a little less than 50 or a little more than 50.

4 Similarly employ 100.

NOTE.—This plan will establish the habit of methodically and promptly discovering the next quotient figure

DESK WORK.

13. Divide:—

1. 725 by 25	4. 8725 by 25	7. 82525 by 25
2. 1225 by 25	5. 42125 by 25	8. 70625 by 25
3. 9825 by 25	6. 64825 by 25	9. 294525 by 25

DESK WORK.

14. 1. Divide 15755 by 23; by 24; by 26.
2. Divide 20736 by 24; by 23; by 26.
3. Divide 19684 by 26; by 28; by 22.
4. Divide 101703 by 29; by 21; by 27
5. Divide 242816 by 28; by 26; by 24

DESK WORK.

15. Divide:—

1. 42800 by 50	6. 29900 by 50	11. 327400 by 50
2. 38400 by 50	7. 14750 by 50	12. 240850 by 50
3. 34350 by 50	8. 282100 by 50	13. 467800 by 50
4. 24650 by 50	9. 341550 by 50	14. 184450 by 50
5. 19450 by 50	10. 438250 by 50	15. 283250 by 50

LESSON 47—CLASS WORK.

16. Divide each of the following numbers by 49; by 51; by 48; by 52; by 47; by 53; by 46; by 54; by 45; by 55; by 43; by 57:

1. 42800	6. 29900	11. 327400
2. 38400	7. 14750	12. 240850
3. 34350	8. 282100	13. 467800
4. 24650	9. 341550	14. 184450
5. 19450	10. 438250	15. 283250

LESSON 48—CLASS WORK.

17. Divide:—

1. 25600 by 100	6. 26700 by 100	11. 635700 by 100
2. 56800 by 100	7. 73400 by 100	12. 245800 by 100
3. 41300 by 100	8. 146800 by 100	13. 538900 by 100
4. 31500 by 100	9. 843500 by 100	14. 967800 by 100
5. 59400 by 100	10. 583200 by 100	15. 798600 by 100

LESSON 49—CLASS WORK.

18. Divide each of the following numbers by 101; by 99; by 102; by 98; by 103; by 97; by 104; by 95; by 96; by 105; by 93; by 108; by 107:—

1. 25600	6. 26700	11. 635700
2. 56800	7. 73400	12. 245800
3. 41300	8. 146800	13. 538900
4. 31500	9. 843500	14. 967800
5. 59400	10. 583200	15. 798600

DESK WORK.

19. Divide:—

1. 804075 by 15	10. 45128624 by 328
2. 915628 by 14	11. 1785062 by 565
3. 134164 by 17	12. 9067435 by 298
4. 164388 by 19	13. 5649852 by 803
5. 127696 by 16	14. 24963185 by 789
6. 125982 by 18	15. 92960243 by 486
7. 111488 by 13	16. 38492738 by 678
8. 1273866 by 22	17. 64382965 by 2153
9. 8568840 by 21	18. 75986410 by 2565

LESSON 50—CLASS WORK.

20. Proof: Multiply the divisor and quotient together and add the remainder, if any, to the product. The result should = the dividend.

21. To divide by 10, 100, 1000, etc., remove the period as many orders to the left as there are ciphers in the divisor. The figures on the left will indicate the quotient and those on the right the remainder.

NOTE —Drill on 15 and 16 until pupils understand how to prove problems in division and how to divide by such divisors as 10, 100, 1000, etc.

LESSON 51—CLASS WORK.

PERCENTAGE.

22. What per cent of a yard makes a yard? A pound is what per cent of itself? A ream is what per cent of itself? What per cent of a peck makes a peck? What per cent of a day makes a day? A gallon is what per cent of a gallon?

Two bushels are what per cent of one bushel? Three bushels are what per cent of one bushel? One bushel is what per cent of a half-bushel? A gallon is what per cent of a gallon? Two gallons are what per cent of one gallon? Three gallons are what per cent of one gallon? A gallon is what per cent of a half-gallon? A yard is what per cent of itself? Three yards are what per cent of one yard? A day is what per cent of twelve hours? A ream is what per cent of ten reams? A peck is what per cent of a bushel? A bushel is what per cent of a peck? A bushel is what per cent of three pecks? A peck is what per cent of a quart? A peck is what per cent of two quarts? A peck is what per cent of three quarts? A peck is what per cent of six quarts? A yard is what per cent of a foot? A yard is what per cent of two feet? A foot is what per cent of three inches? A foot is what per cent of four inches?

LESSON 52—CLASS WORK.

23. What is 10 % of 100 yards?
What is 10 % of 200 yards?
What is 10 % of 600 yards?
What is 25 % of 100 yards?
What is 75 % of 100 yards?
What is $66\frac{2}{3}$ % of 100 yards in yards and feet?
What is 25 % of 600 yards?
What is 50 % of 1000 gallons?
What is 25 % of 1000 gallons?
What is $12\frac{1}{2}$ % of 1000 gallons?
What is $6\frac{1}{4}$ % of 1000 gallons?
What is 75 % of 1000 gallons?

24. Drills to be placed on chart or blackboard:

1. $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$ = % of $\frac{1}{2}$

2. $\frac{1}{2}$ = % of $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$

3. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{10}$, $\frac{2}{5}$, $\frac{4}{5}$, $\frac{1}{30}$, $\frac{2}{15}$, $\frac{1}{6}$, $\frac{4}{15}$, $\frac{9}{30}$ = % of $\frac{1}{3}$

4. $\frac{1}{3}$ = % of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{10}$, $\frac{2}{5}$, $\frac{4}{5}$, $\frac{1}{30}$, $\frac{2}{15}$, $\frac{1}{6}$, $\frac{4}{15}$, $\frac{9}{30}$

CHAPTER XXIII.

Analysis—Percentage.

NOTE.—Pupils at this stage of advancement should learn to analyze simple problems in addition, subtraction, multiplication, and division. The following list of one hundred problems is intended for this purpose Similar supplementary questions may be given by the teacher if necessary.

LESSON 53—CLASS WORK.

PROBLEMS.

1. A lady spent 25 cents for berries, 15 cents for meat, 10 cents for bread, and 12 cents for milk; what did she spend for all?

2. How many yards of carpet in three strips if the first contains 25 yards, the second 30, and the third 40 yards?

3. How many feet of rope will it take to go around a lot 60 feet long and 25 feet wide?

4. A man paid $20.00 for a suit, $5.00 for shoes, $3.00 for a hat, and $22.00 for groceries; what did he pay for all?

5. A milk-man has six cans in which he takes milk to market; the smallest holds 12 quarts, the next in size holds 14 quarts, the next 16 quarts, the next 18 quarts, the next 20 quarts and the largest 24 quarts; how many quarts do they all hold?

6. A teacher consumed 17 minutes in hearing a spelling lesson, 19 minutes in hearing a music lesson, 29 minutes in hearing a reading lesson, 27 minutes in hearing a grammar lesson, 32 minutes in hearing an arithmetic lesson, and 28 minutes in hearing a history lesson; how many minutes was she in hearing all?

7. A bicyclist rides the first lap in 55 seconds, the second in 54 seconds, the third in 58 seconds, the fourth in 49 seconds, and the fifth in 57 seconds; how long did it take him to ride the five laps?

8. A grocer bought 23 gallons of vinegar, 32 gallons of coal oil, 47 gallons of molasses, and 28 gallons of cider; how many gallons in all did he buy?

9. The several shelves of a book-case contain respectively 28, 32, 19, 26, and 37 books; how many books in the case?

10. A man traveled 64 miles on Monday, 85 miles on Tuesday, 19 miles on Wednesday, 48 miles on Thursday, 52 miles on Friday, 76 miles on Saturday; how far did he travel in the six days?

11. If a piece of land contains 236 acres, another 348 acres, another 569 acres, another 197 acres, and another 486 acres, how much do they all contain?

12. A farmer took to market a load of watermelons each week for five consecutive weeks; the first load had in it 76 melons, the second 85, the third 148, the fourth 137, and the fifth 99; how many did he market in all?

13. A, B, C, D, E, and F weigh respectively 147, 156, 173, 188, 169, and 137 pounds; what is their combined weight?

Lesson 54—Class Work.

14. A milk-man has six milk cans; one will hold 95 pints of milk, another 115 pints, another 176 pints, another 192 pints, another 128 pints, and another 145 pints; how much will they all hold?

15. Eight 25-foot lots are respectively 145, 137, 148, 129, 138, 142, 165, and 156 feet in length; how long must a piece of ground be to equal their combined length?

16. In a barrel there are 196 pounds of flour; in another 187 pounds; in another 179 pounds, in another 166 pounds, in another 57 pounds; in another 68 pounds, and in another 149 pounds; how many pounds of flour in all?

17. A man bought six lots of pork, the first containing 236 barrels, the second, 379 barrels; the third, 467 barrels; the fourth, 198 barrels; the fifth, 725 barrels, and the sixth, 588 barrels. How many barrels did he buy in all?

18. A farmer raised in one year 2765 bushels of corn, 1987 bushels of wheat, 1568 bushels of oats, 2149 bushels of rye, and 1657 bushels of barley; how many bushels of grain in all did he raise?

19. Several rugs cover, in order of their size, 1296 square inches, 1512 square inches, 1584 square inches, 1728 square inches, 2016 square inches, and 2592 square inches; how many square inches do they all cover?

20. A roll of carpet has 45 yards in it; another, 65 yards; another, 115 yards; another, 138 yards; another, 149 yards. How many yards in all?

21. A school building has 7388 square feet of floor space; another, 14776; another, 32164; another, 9845; another, 12879; another, 18567. What number of square feet of floor space in all?

22. A school building has in it 2164 square yards of plastering; another, 4328; another, 6492; another, 3827; another, 5796; another, 8566. How many square yards have they all?

23. A laborer made eight excavations. From the first he took 1233 cubic feet of earth; from the second, 1823 cubic feet; from the third, 2417 cubic feet; from the fourth, 2618 cubic feet; from the fifth, 3027 cubic feet; from the sixth, 3592 cubic feet; from the seventh, 3823 cubic feet, and from the eighth, 4369 cubic feet. How large a mound can be built with the earth from all?

24. A wind-mill on an Oklahoma ranch pumped water continuously for four weeks. The first week it pumped 4893 gallons of water; the second, 5863 gallons; the third, 6822, and the fourth, 6973 gallons. How many gallons did it pump in all?

25. The superintendent of buildings in a certain city issued during one week permits for buildings to cost respectively: $111,123, $113,153, $210,425, $212,853, $856,312, $167,893, and $567,987. What was the aggregate cost of all?

Lesson 55—Class Work.

26. A merchant sells suits for $25.00 each that cost him $15 each. How much does he gain a suit?

27. Frank went to the store with 95 cents; after spending 75 cents, how much had he left?

28. A grocer sold 35 pounds of a sack of beans containing 83 pounds; how many pounds of beans were left in the sack?

29. From a barrel of vinegar containing 41 gallons 10 gallons leaked out; how many gallons were left in the barrel?

30. A stonemason has 56 minutes in which to complete a wall; he did so in 17 minutes less time; how long did it take him?

31. A painter bought 31 gallons of turpentine and lost 12 gallons by leakage; how many gallons did he have remaining?

32. Henry picked 91 quarts of cherries in two days; George picked 74 quarts in the same time; how many more quarts did Henry pick than George?

33. A druggist had on his shelves 193 pint bottles of perfumery; after selling 93 pints how many pints had he left?

34. A full barrel of flour contains 196 pounds; what does a barrel containing 167 pounds lack of being full?

35. The surface of a table contains 576 square inches. How much larger is it than one containing only 197 square inches?

36. Fifteenth street is 331 rods long and 56 rods longer than Twelfth street. How long is Twelfth street?

37. The shaft of a coal mine is 375 feet deep. A platform 296 feet from the bottom is how many feet from the top?

LESSON 56—CLASS WORK.

38. A factory received an order for 937 peck measures. The firm lacked 198 measures of having enough to fill the order. How many did it have on hand?

39. Find the length of a wall that is shorter by 193 inches than a tape line measuring 480 inches?

40. A farmer having 686 acres sold 548 acres. How many acres did he have left?

41. A private library contained 1338 books; 249 were destroyed or damaged by fire. How many books were left uninjured?

42. A farmer had 1298 bushels of wheat. After selling 699 bushels, how many bushels had he left?

43. A grocer bought four casks of sugar containing 1895 pounds. After selling 899 pounds how many had he left?

44. A man bought five lots for $5,124; he sold them for $7,023. How much did he gain?

45. I owe A and B together $5,775. If I owe B $2,876, how much more do I owe A than B?

46. A farmer raised 3978 bushels of corn. He raised 1733 more bushels of corn than he did of wheat. How much wheat did he raise?

47. A's ranch contains 3734 acres less than B's. In B's ranch there are 11200 acres. How many acres are there in A's ranch?

48. A's income is $16,200. His expenses are $9,312. How much has he left at the end of the year?

49. A dealer had on hand 11365 barrels of sugar of which he sold 9708. How many had he left?

50. A passenger locomotive ran 122743 miles in two years. A freight locomotive ran 73955 miles in the same length of time. How many miles further did the passenger locomotive run than the freight locomotive?

Lesson 57—Class Work.

51. What will five suits of clothes cost at $16.00 a suit?

52. What will be the cost of 15 pounds of nails at 5 cents a pound?

53. Find the cost of six gallons of gasoline at 16 cents a gallon.

54. Walking 24 miles a day, how far will a man walk in three days?

55. A man travels 47 miles a day. How far can he travel in 7 days?

56. There are 144 square inches in a square foot. How many square inches in a square yard?

57. How many pints in 97 gallons?

58. How many inches in 35 yards?

59. How many pounds in 9 bushels of rye?

60. How many ounces in 85 pounds?

61. How many square feet in a section of pavement 285 feet long and 69 feet wide?

62. How many hours in the month of February, this year?

63. There are 320 rods in one mile. How many rods in 97 miles?

Lesson 58—Class Work.

64. A factory has a capacity of 237 barrels of flour. What is the output in a year allowing 312 days to the year?

65. What is the value of a farm of 673 acres worth $67 an acre?

66. If one gas jet will burn an average of 2715 cubic feet in one month, how many cubic feet will be required to keep a building lighted with 37 gas jets?

67. If a paving company lays 2235 square yards of asphalt in one day, how much can it lay in 37 days?

68. A stove factory sold 1897 stoves at an average price of $37 each. What was the total amount received?

69. A coal company sells 5678 car-loads of coal at $137 a car. How much was received for the coal?

70. A stockman sold 1967 head of cattle at $47 per head. What amount did he receive for the cattle?

71. In one mile there are 5280 feet; how many feet in 68 miles?

72. Find the cost of 857 pianos at $475 each.

73. There are 1440 minutes in one day. How many minutes in 365 days?

74. Find the weight of 5637 cattle that have an average weight of 1367 pounds.

75. Find the cost of building 567 miles of railroad at an average cost of $8,765 a mile.

LESSON 59—CLASS WORK.

76. If 5 pounds of beans cost 20 cents what is the cost of 1 pound?

77. If six quarts of nuts are worth 90 cents, what is one quart of nuts worth?

78. How many yards of velvet carpet at $3.00 a yard can be purchased for $75.00?

79. If Henry can earn $7 a week, in how many weeks can he earn $84.00?

80. If a man can travel 175 miles in five days, how far can he travel in one day?

81. If a quart of cranberries cost 15 cents, how many quarts will $3.45 buy?

82. How many bushels in 544 quarts of grass seed?

83. How many pounds in 176 ounces?

84. How many yards in 2592 inches?

85. How many days in 744 hours?

86. How many square feet in 2160 square inches?

87. There are 320 rods in a mile; how many miles in 8320 rods?

88. A has $2,912 with which to buy a farm; how many acres can be purchased at $112 per acre?

LESSON 60—CLASS WORK.

89. A mill turns out in 144 days 9548 barrels of flour. What is the daily average output?

90. A schoolroom contains 7560 cubic feet of air; how many cubic yards of air does it contain?

91. In a train load of calves there are 990. If each car contains 55 head, how many cars in the train?

92. A stockman ships 476 head of cattle in 17 cars; how many cattle in each car?

93. A coal dealer paid $8,010 for 1335 tons of coal. What was the coal per ton?

94. A farm of 285 acres cost $18,525; what was the cost per acre?

95. There are 5280 feet in a mile; how many rails 60 feet long will it take to lay one side of a single railway track this length?

96. A steamship made a trip of 2432 miles in eight days; how far did it travel each day?

97. How many days would it require a ship to make a trip of 4864 miles, traveling 304 miles a day?

98. A lake containing 92,292 gallons of water was pumped dry in 67 days. What was the average number of gallons taken out per day?

99. The private soldiers of an army were paid a total of $4,706,455. If the number of men was 5935, how much did each receive?

100. The population of Missouri is 2,679,184, and its area 68,735 square miles. What is the average population to the square mile?

Lesson 61—Class Work.

PERCENTAGE.

1. 1728 cubic inches = 1 cubic foot.

What is $\frac{1}{2}$ of a cubic foot in cubic inches?
What is $\frac{1}{4}$ of a cubic foot in cubic inches?
What is $\frac{1}{8}$ of a cubic foot in cubic inches?
What is $\frac{1}{16}$ of a cubic foot in cubic inches?
What is $\frac{1}{32}$ of a cubic foot in cubic inches?
What is $\frac{1}{3}$ of a cubic foot in cubic inches?
What is $\frac{1}{6}$ of a cubic foot in cubic inches?
What is $\frac{1}{12}$ of a cubic foot in cubic inches?
What is $\frac{1}{24}$ of a cubic foot in cubic inches?
What is 50 % of a cubic foot in cubic inches?
What is 25 % of a cubic foot in cubic inches?
What is $12\frac{1}{2}$ % of a cubic foot in cubic inches?
What is $6\frac{1}{4}$ % of a cubic foot in cubic inches?
What is $3\frac{1}{8}$ % of a cubic foot in cubic inches?
What is $33\frac{1}{3}$ % of a cubic foot in cubic inches?
What is $16\frac{2}{3}$ % of a cubic foot in cubic inches?
What is $8\frac{1}{3}$ % of a cubic foot in cubic inches?
What is $4\frac{1}{6}$ % of a cubic foot in cubic inches?

Lesson 62—Class Work.

2. 5280 feet = 1 mile.

$\frac{1}{2}$ mile = feet = % of a mile.
$\frac{1}{4}$ mile = feet = % of a mile.
$\frac{1}{8}$ mile = feet = % of a mile.
$\frac{1}{16}$ mile = feet = % of a mile.
$\frac{1}{32}$ mile = feet = % of a mile.
$\frac{1}{3}$ mile = feet = % of a mile.
$\frac{1}{6}$ mile = feet = % of a mile.
$\frac{1}{12}$ mile = feet = % of a mile.
$\frac{1}{24}$ mile = feet = % of a mile.

LESSON 63—CLASS WORK.

3. What is 50 % of $3200?
What is $12\frac{1}{2}$ % of $3200?
What is 10 % of $3200?
What is 20 % of $3200?
What is 80 % of $3200?
What is 60 % of $3200?
What is 40 % of $3200?
What is 25 % of $3200?
What is 75 % of $3200?
What is $6\frac{1}{4}$ % of $3200?
What is $3\frac{1}{8}$ % of $3200?
What is 150 % of $3200?
What is 200 % of $3200?

THE END.